Functional Analysis F3/F4/NVP (2005)
Remark on Homework assignment 2, problem 4 (§4.3: 14)

To clarify (and not make the problem unnecessarily difficult):

1. You should assume that X is a *real* normed space (ie $K = \mathbb{R}$).

2. You may take the facts stated in problem §2.8: 15 (p. 111) as the definition of *hyperplane*, and the two *half spaces* determined by it. In other words, by *hyperplane* we mean any set

$$H = \{x \in X \mid f(x) = c\},$$

for some arbitrary, fixed $c \in \mathbb{R}$ and $f \in X'$. The two *half spaces* determined by this hyperplane H are:

$$X_1 = \{x \mid f(x) \leqq c\} \qquad \text{and} \qquad X_2 = \{x \mid f(x) \geqq c\}.$$