

Note that the document on <http://www.math.uu.se/~astrombe/analtalt08/analtalt08.html> is always the most updated version.

1. PRIMES IN ARITHMETIC PROGRESSIONS

p.9: In proof of Remark 1.2, I corrected “Fermat’s Little Theorem” into “Euler’s Theorem”.

p.9: In Remark 1.3, I corrected all “ $q - 1$ ” into “ $\phi(q)$ ”.

Note that formula (25) on p. 13 is correct as written; I got confused in class.

(Also note that regarding formulae (6) and (21), it is important to mention that we choose the *principal branch* of each logarithm in the right hand side; otherwise the right hand side would be just as multivalued as the left hand side. This is written in the notes, but I forgot to say it during my lecture.)

2. INFINITE PRODUCTS

I added a 5-line parenthesis just below (52) to clarify a point which I managed to get confused over in class.

3. PARTIAL SUMMATION AND DIRICHLET SERIES

(During the lecture I claimed, in connection with Theorem 3.11, that

$$\limsup_{x \rightarrow \infty} \frac{\log |A(x)|}{\log x} = \inf \{ \theta \in \mathbb{R} : |A(x)| \ll x^\theta, \forall x \geq 1 \}.$$

I’ve now added a Problem, 3.12, to prove this. Note this could be called an exercise belonging to “Foundations of real analysis”.)

p.43, (114) and (116): Corrected by inserting a minus sign in the right hand side of each formula.

p.44, Problem 3.9: “ $\sigma > 1 + \operatorname{Re} \alpha$ ” corrected to “ $\sigma > \max(1, 1 + \operatorname{Re} \alpha)$ ”.

4. MORE ON DIRICHLET CHARACTERS

At the beginning of section 4 I added some recommended textbooks concerning the structure of $(\mathbb{Z}/q\mathbb{Z})^\times$ and quadratic reciprocity.

5. $L(1, \chi)$ AND CLASS NUMBERS

p.97, just above (227): I added a reference to Theorem I.6.3 in Katznelson’s book to justify the absolute convergence of the Fourier series.

p.97(227): The parenthesis should read “(with $\widehat{\varphi}(m) = \dots$)” in place of the incorrect “(with $\varphi(m) = \dots$)”.

6. THE DISTRIBUTION OF PRIMES

7. THE PRIME NUMBER THEOREM

p.131, problem 7.4(a): “ dx ” corrected to “ ds ” in the integral formula for $M_1(x)$.

8. THE Γ -FUNCTION; INTEGRAL FUNCTIONS OF ORDER 1

p.133, lines 7-8: replaced “ $\log B + R_m^\alpha$ ” with “ $\max(0, \log B + R_m^\alpha)$ ”. This is to make the statements correct also if $\log B + R_m^\alpha < 0$. (Note that this does not cause any changes in the later discussion where we let $m \rightarrow \infty$, for then $R_m \rightarrow \infty$, and thus eventually we have $\max(0, \log B + R_m^\alpha) = \log B + R_m^\alpha$.)

p.134, end of proof of Prop. 8.3: We should write lower limit “0+” in place of 0 in the first integral, to make it well-defined according to Definition 3.1 (and Def 3.2); the problem is that the requirement $g \in C([A, B])$ is not true for $g(r) = \log(R/r)$ and $[A, B] = [0, R]$! The computation is correct with “0+” since we anyway have $n(r) = 0$ for r sufficiently small.

p.136, two lines above (306): Perhaps slightly more appropriate to write “the intervals $[r_n - r_n^{-\alpha}, r_n + r_n^{-\alpha}]$ ” in place of “the intervals $(r_n - r_n^{-\alpha}, r_n + r_n^{-\alpha})$ ” (although the total length of course is the same for both cases).

p.137, lines 2-5: The claim “ $|P_2(z)| \geq (C_5 R^{-1-\alpha})^{C_6 R^\alpha}$ ” would in general be false if $C_5 > 1$. I’ve fixed this by assuming $C_5 < 1$ in line 3, and also added a footnote discussing the deduction.

p.144, last line: Sign corrected in the Taylor expansion.

p.147, (330): In the last integral, “ $F^{(h)}(x)$ ” should be “ $f^{(h)}(x)$ ”.

9. THE FUNCTIONAL EQUATION

10. THE INFINITE PRODUCTS FOR $\xi(s)$ AND $\xi(s, \chi)$

p.161, in the proof of Proposition 10.1, I removed the false claim that “ $\zeta(s)$ is bounded in the compact rectangle $\{\frac{1}{2} \leq \sigma \leq 2, |t| \leq 1\}$ ”. Note that the conclusion in the next sentence, that $|\zeta(s)| \leq e^{C_3|s|}$ for $\sigma \geq \frac{1}{2}$ and $|s|$ large, is still valid.

11. ZERO-FREE REGIONS FOR $\zeta(s)$ AND $L(s, \chi)$

p.176, (411): “ $0 < \gamma < \dots$ ” corrected to “ $0 \leq \gamma < \dots$ ”.

12. THE NUMBERS $N(T)$ AND $N(T, \chi)$ 13. THE EXPLICIT FORMULA FOR $\psi(x)$

p.209, (474): Two “1” corrected to “ $\log x$ ”. Also in (475) and in line 2 of Remark 13.2, “1” has been corrected to “ $\log x$ ”.

14. THE EXPLICIT FORMULA FOR $\psi(x, \chi)$

15. THE PRIME NUMBER THEOREM FOR ARITHMETIC PROGRESSIONS (I)

16. SIEGEL’S THEOREM

17. THE POLYA-VINOGRADOV INEQUALITY

18. FURTHER PRIME NUMBER SUMS

p.249, (579): The second summation corrected to “ $\sum_{M < m \leq \min(N/V, 2M)}$ ” (viz., one “ M ” corrected to “ $2M$ ”).

19. SUMS OF THREE PRIMES

20. THE LARGE SIEVE

21. BOMBIERI'S THEOREM