**Uppsala universitet**
**Matematiska institutionen**
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**Analytisk talteori, 1MA038**
**Tentamen den 8 januari 2018**

*Inga hjälpmendel (Just paper and pen. No calculators, books, or notes etc.)*

**1.** Define: a) Euler's partition function $p(n)$, b) Dirichlet convolution. And c) give the statement of the prime number theorem.

**2.** Formulate and prove the Euler product formula for Riemann's zeta funtion.

**3.** Prove that the Dirichlet series with the Möbius function $\mu(n)$ as coefficients equals $1/\zeta(s)$, the reciprocal of the Riemann zeta function.

**4.** Consider primes on the form $4k+1$ or on the form $4k+3$. In one of these cases prove that there are infinitely many primes on that form using a modification of the proof in Euclid concerning the infinitude of prime numbers.

**5.** Define and provide a meromorphic continutation to all of $\mathbb{C}$ of the arithmetic function $n \mapsto n!$.

**6.** Let $k > 1$ be an integer. Let $\chi$ be any non-principal Dirichlet character (mod $k$). For any positive integers $a < b$ consider

$$A(a,b) = \left| \sum_{n=a}^{b} \chi(n) \right| .$$

Let $\varphi$ denote Euler's (totient) function. As a warm-up prove that $A(a,b) \leq \varphi(k)$. Could this inequality be improved to $A(a,b) \leq \frac{1}{2}\varphi(k)$? Give a proof or a counterexample.