# Finitary 2-representations of finitary 2-categories

Volodymyr Mazorchuk

(Uppsala University)

Workshop "Algebraic Lie theory and representation theory"
September 2, 2014, Edinburgh, UK

# Coauthor

This is a report on a joint project with

Vanessa Miemietz

from University of East Anglia

# Motivating example 1.

$\mathfrak{g}$ — semi-simple finite dimensional complex Lie algebra

$\mathfrak{g} = \mathfrak{n}_- \oplus \mathfrak{h} \oplus \mathfrak{n}_+$ — fixed triangular decomposition

$\mathcal{O}$ — BGG category $\mathcal{O}$

$\mathcal{O}_0$ — the principal block of $\mathcal{O}$

$\mathcal{P} : \mathcal{O}_0 \to \mathcal{O}_0$ — the 2-category of projective endofunctors of $\mathcal{O}_0$

Fact: $\mathcal{P}$ has finitely many indecomposables up to isomorphism

Fact: $\mathcal{P}$ has finite dimensional spaces of 2-morphisms

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$\mathfrak{G}$ — 2-Kac-Moody algebra of finite type

$\mathcal{L}(\lambda)$ — simple highest weight 2-representation of $\mathfrak{G}$

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$A$ — finite dimensional $\Bbbk$-algebra

Definition: A projective endofunctor of $A$-mod is tensoring with a projective $A$–$A$-bimodule, up to isomorphism

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# Finitary $\Bbbk$-linear categories.

Definition: An additive $\Bbbk$-linear category $\mathcal{A}$ is finitary if

- $\mathcal{A}$ is idempotent split;
- $\mathcal{A}$ has finitely many indecomposables;
- all morphism spaces in $\mathcal{A}$ are finite dimensional (over $\Bbbk$).

Example: $A$-proj for a finite dimensional $\Bbbk$-algebra $A$.

Fact: If $\mathcal{A}$ is finitary, then $\mathcal{A} \cong A$-proj for some $A$.

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Definition: A 2-category $\mathcal{C}$ is finitary over $\Bbbk$ if

- $\mathcal{C}$ has finitely many objects;
- each $\mathcal{C}(\mathtt{i}, \mathtt{j})$ is finitary $\Bbbk$-linear;
- composition is biadditive and $\Bbbk$-bilinear;
- identity 1-morphisms are indecomposable.

Examples:

- Projective functors on $\mathcal{O}_0$;
- Soergel bimodules over the coinvariant algebra;
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# Combinatorics of finitary 2-categories.

$\mathcal{C}$ — finitary 2-category

$\Sigma(\mathcal{C})$ — isoclasses of indecomposable 1-morphisms in $\mathcal{C}$

Fact: $\Sigma(\mathcal{C})$ is a multisemigroup under

$$\mathrm{F} \star \mathrm{G} = \{\mathrm{H} : \mathrm{H} \text{ is isomorphic to a direct summand of } \mathrm{FG}\}$$

Left preorder: $\mathrm{F} \geq_L \mathrm{G}$ if $\mathrm{F} \in \Sigma(\mathcal{C})\mathrm{G}$

Left cells: equivalence classes w.r.t. $\geq_L$ (a.k.a. Green's $\mathcal{L}$-classes)

Similarly: right and two-sided preorders $\geq_R$ and $\geq_J$ and right and two-sided cells

Example: For Soergel bimodules (projective functors on $\mathcal{O}_0$) these are Kazhdan-Lusztig orders and cells

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Similarly: right and two-sided preorders $\geq_R$ and $\geq_J$ and right and two-sided cells

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$A$ — basic, connected finite dimensional $\Bbbk$-algebra

$1 = e_1 + e_2 + \cdots + e_n$ — primitive decomposition of $1 \in A$

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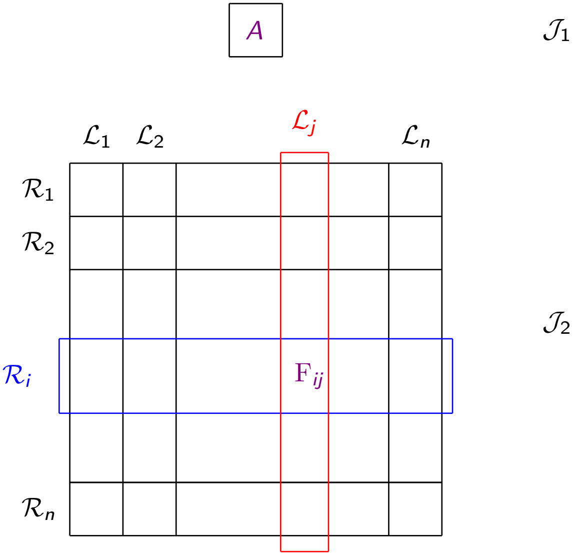

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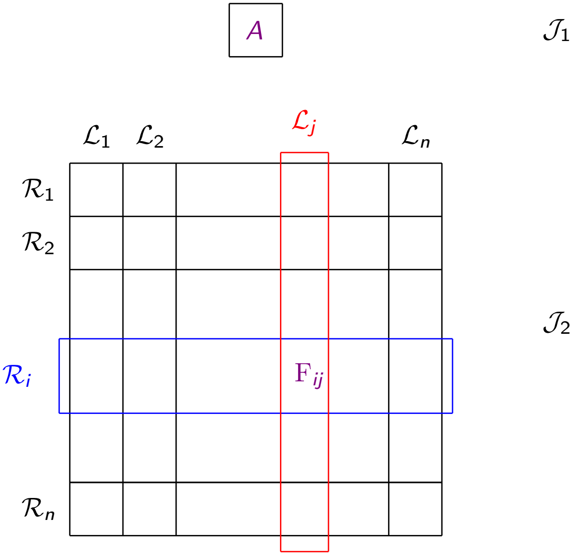

# Strongly regular two-sided cells

Notable properties of $\mathcal{J}_2$ in the above example:

- different left cells inside $\mathcal{J}_2$ are not $\geq_L$-comparable;
- different right cells inside $\mathcal{J}_2$ are not $\geq_R$-comparable;
- $|\mathcal{L}_j \cap \mathcal{R}_i| = 1$

Definition: A two-sided cell $\mathcal{J}$ having the above properties is called strongly regular

Examples:

- both 2-sided cells in $\mathcal{C}_A$
- all 2-sided cells in $\mathfrak{G}_\lambda$
- all two-sided cells for Soergel bimodules (proj. functors) in type $A$

Note: Fails for Soergel bimodules outside type $A$ in general

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# 2-representations

$\mathcal{C}$ — finitary 2-category

"Definition": A 2-representation of $\mathcal{C}$ is a functorial action of $\mathcal{C}$ on a suitable category(ies).

Example: Principal 2-representation $\mathbf{P}_{\mathtt{i}} := \mathcal{C}(\mathtt{i}, \_)$ for $\mathtt{i} \in \mathcal{C}$

Note: 2-representations of $\mathcal{C}$ form a 2-category where

- 1-morphisms are 2-natural transformations
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# Cell 2-representations

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$\mathcal{L}$ — left cell in $\mathcal{C}$

$\mathtt{i}$ — the source for 1-morphisms in $\mathcal{L}$

$\mathbf{P}_{\mathtt{i}}$ — the $\mathtt{i}$-th principal 2-representation

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**M** — 2-representation of $\mathcal{C}$

Definition: **M** is finitary if $\mathbf{M}(\mathtt{i})$ is finitary $\Bbbk$-linear for all $\mathtt{i}$

Definition: **M** is transitive if **M** is finitary and for any indecomposable $X, Y$ in **M** there is a 1-morphism $\mathrm{F}$ such that $X$ is isomorphic to a direct summand of $\mathrm{F}\,Y$

Intuition: Transitive action of a group (for us: a multisemigroup)

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Theorem. [M-Miemietz]:

Every "simple" fiat 2-category with a strongly regular maximal two-sided cell is "essentially" $\mathcal{C}_A$ for $A$ self-injective and weakly symmetric.

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# Simple transitive 2-representations

Theorem. [M-Miemietz]:

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# Inflation

$\mathcal{C}$ — finitary 2-category

$\mathbf{M}$ — finitary 2-representation of $\mathcal{C}$

$\mathcal{A}$ — finitary $\Bbbk$-linear category

$\mathbf{M} \boxtimes \mathcal{A}$ has the structure of a 2-representation of $\mathcal{C}$ where everything in $\mathcal{C}$ acts as the identity on $\mathcal{A}$

Definition: This is called the inflation of $\mathbf{M}$ by $\mathcal{A}$

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# Isotypic 2-representations

Recall: An $A$-module is isotypic if all its simple subquotients are isomorphic

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# Classification of isotypic 2-representations

Main theorem. [M-Miemietz]

$\mathcal{C}$ — fiat 2-category with unique maximal two sided cell $\mathcal{J}$

Assume every non-trivial 2-sided ideal in $\mathcal{C}$ contains $\mathcal{J}$

Assume $\mathcal{J}$ is strongly regular.

Then every faithful isotypic 2-representation of $\mathcal{C}$ is equivalent to an inflation of $\mathbf{C}_{\mathcal{L}}$ for a left cell $\mathcal{L}$ in $\mathcal{J}$.

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THANK YOU!!!