Simple transitive 2-representations of finitary 2-categories

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The aim of the talk

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Let $\mathcal{C}$ be a 2-category.

Main problems of the 2-representation theory of $\mathcal{C}$

- Understand the 2-category of all 2-representation of $\mathcal{C}$
- Understand all "simple" 2-representation of $\mathcal{C}$
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$\Bbbk$ — algebraically closed field

Definition. A 2-category $\mathscr{C}$ is called finitary provided that

- $\mathscr{C}$ has finitely many objects
- Each $\mathscr{C}(\mathtt{i},\mathtt{j})$ is additive, idempotent split, Krull-Schmidt and has finitely many indecomposables
- 2-morphisms form finite dimensional $\Bbbk$-vector spaces
- compositions are additive and $\Bbbk$-linear
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Example of a finitary 2-category

A — finite dimensional, basic, connected non-semi-simple $\Bbbk$-algebra

Definition. The 2-category $\mathcal{C}_A$ is defined as follows

- $\mathcal{C}_A$ has one object (identified with A-mod)
- 2-morphisms in $\mathcal{C}_A$ are endofunctors of A-mod isomorphic to a direct sum of copies of the identity functor and functors of tensoring with projective A-bimodules
- 2-morphisms in $\mathcal{C}_A$ are natural transformations of functors

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Absolutely simple finitary 2-category

Definition. A finitary 2-category $\mathcal{C}$ is called absolutely simple provided that

- $\mathcal{C}$ has one object
- given any indecomposable 1-morphisms F and G which are not isomorphic to the identity 1-morphism, there exist 1-morphisms H and K such that F is isomorphic to a direct summand of HGK

Informally. The two-sided ideal generated by G contains F

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Theorem. [M.-Miemietz] Roughly speaking (i.e. under some mild technical assumptions) every absolutely simple finitary 2-category is biequivalent to $\mathcal{C}_A$ for an appropriate self-injective A

Consequence. The finitary 2-category $\mathcal{C}_A$ for a self-injective A is a "proper" categorification of the simple algebra $\mathrm{Mat}_{n\times n}(\mathbb{C})$

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$\mathcal{C}$ — finitary 2-category

Definition. A finitary 2-representation of $\mathcal{C}$ is a 2-functors $\mathbf{M}$ which assigns

- to every object of $\mathcal{C}$ an additive, idempotent split, Krull-Schmidt category with finitely many indecomposables
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Example. $\mathcal{C}(\mathtt{i}, \_)$, the so-called principal 2-representation

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Definition. $\mathbf{M}$ is transitive provided that for any indecomposable objects $X \in \mathbf{M}(\mathtt{i})$ and $Y \in \mathbf{M}(\mathtt{j})$ there is a 1-morphism $\mathrm{F} \in \mathcal{C}(\mathtt{i}, \mathtt{j})$ such that Y is isomorphic to a direct summand of $\mathbf{M}(\mathrm{F})\, X$.

Intuitive example. Transitive action of a group

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Definition. $\mathbf{M}$ is simple transitive if $\mathbf{I} = 0$

Example. If $\mathbf{M}$ is transitive, then $\mathbf{M}/\mathbf{I}$ is simple transitive

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Part III

Recall:

Fact. Every simple finite dimensional module over $\mathrm{Mat}_{n\times n}(\mathbb{C})$ is isomorphic to the natural module $\mathbb{C}^n$

In this part we categorify:

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Cells

$\mathcal{C}$ — finitary 2-category

$\mathcal{S}[\mathcal{C}]$ — set of isoclasses of indecomposable 1-morphisms (with 0)

$\mathcal{S}[\mathcal{C}]$ — multisemigroup (induced by composition)

Definition of left preorder. For $\mathrm{F}, \mathrm{G} \in \mathcal{S}[\mathcal{C}]$ set $\mathrm{F} \geq_L \mathrm{G}$ if there is $\mathrm{H} \in \mathcal{S}[\mathcal{C}]$ such that F is isomorphic to a direct summand of HG.

Similarly: Right preorder $\geq_R$ and two-sided preorder $\geq_J$.

Definition. Classes of equivalent elements for these preorders are called cells (left, right or two-sided)

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$\mathcal{L}$ — left cell in $\mathcal{S}[\mathcal{C}]$

$\overline{\mathcal{L}}$ — the coideal w.r.t. $\geq_L$ generated by $\mathcal{L}$

Note: All elements from $\overline{\mathcal{L}}$ start at some $\mathtt{i} \in \mathcal{C}$

$\mathbf{M}_1$ — the additive closure of $\overline{\mathcal{L}}$ in $\mathcal{C}(\mathtt{i}, \_)$

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Note: Indecomposables in $\mathbf{N}$ are in bijection with $\mathcal{L}$

Note: $\mathbf{N}$ is transitive!

Definition: The cell 2-representation $\mathbf{C}_{\mathcal{L}}$ is defined as the simple transitive quotient of $\mathbf{N}$

Example:

- A — self injective, connected, basic finite dimensional $\Bbbk$-algebra
- $1 = e_1 + e_2 + \cdots + e_n$, e_i — primitive idempotents, $e_i e_j = \delta_{i,j}$
- $\mathcal{C}_A$ — the corresponding 2-category
- $\mathcal{L} = \{Ae_j \otimes e_1 A : j = 1, 2, \ldots, n\}$

Claim: The cell 2-representation $\mathbf{C}_{\mathcal{L}}$ is equivalent to the defining 2-representation (i.e. action of $\mathcal{C}_A$ on A-mod)

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Other examples.

- For the 2-category of Soergel bimodules in type A cell 2-representations are minimal categorifications of Specht modules
- For the 2-category of Soergel bimodules in other types cell 2-representations are minimal categorifications of Kazhdan-Lusztig cell modules
- For finitary quotients of 2-Kac-Moody algebras (of finite type) cell 2-representations are minimal categorifications of simple finite dimensional modules

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Main result

We want to categorify:

Fact. Every simple finite dimensional module over $\mathrm{Mat}_{n\times n}(\mathbb{C})$ is isomorphic to the natural module $\mathbb{C}^n$

Theorem.[M.-Miemietz] Let A be a finite dimensional, basic, connected, non-semi-simple and self-injective $\Bbbk$-algebra. Then every simple transitive 2-representation of $\mathcal{C}_A$ is equivalent to a cell 2-representation

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Note. The latter corollary covers Soergel bimodules in type A and finitary quotients of 2-Kac-Moody algebras

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$$(A \otimes_{\Bbbk} A) \otimes_A (A \otimes_{\Bbbk} A) \cong (A \otimes_{\Bbbk} A)^{\dim A}$$

$\mathrm{F} := A \otimes_{\Bbbk} A$

$\mathbf{M}$ — simple transitive 2-representation of $\mathcal{C}_A$

$[\mathrm{F}]$ — the matrix of $\mathbf{M}(\mathrm{F})$

Observation. $[\mathrm{F}]$ is a positive matrix, $[\mathrm{F}]^2 = (\dim A) \cdot [\mathrm{F}]$

Use Perron-Frobenius Theorem to estimate the Cartan matrix of the algebra underlying $\mathbf{M}(A\text{-}\mathrm{mod})$

Compare this estimate to the Cartan matrix of A and use inclusion of algebra to establish an isomorphism

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THANK YOU!!!