UPPSALA UNIVERSITET
MATEMATISKA INSTITUTIONEN
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Complex Analysis 10hp
Kand Ma2
Spring Term 2014

Home assignment 1

Note: Solutions to the problems should contain both detailed arguments and calculations (where appriopriate).

1. Let C be a generalised circle in the Riemann sphere $\hat{\mathbb{C}}$. Let z_1 and z_2 be points belonging to the circle C and z_3 a point in $\hat{\mathbb{C}}$ which does not belong to C. Show that there exists a Möbius transformation T which maps the circle C to itself, maps the point z_1 to z_2 and has z_3 as a fix-point. How many such Möbius transformations do exist? (Prove all your claims carefully!)

2. Show that if z_1, z_2, z_3, z_4 are distinct points in the extended complex plane and T is a Möbius (linear fractional) transformation then $(Tz_1, Tz_2, Tz_3, Tz_4) = (z_1, z_2, z_3, z_4)$.

3. Show that the cross-ratio (z_1, z_2, z_3, z_4) is real if and only if the four points z_1, z_2, z_3, z_4 lie on a circle or on a straight line.

4. Suppose that a Möbius transformation carries one pair of concentric circles onto another pair of concentric circles. Prove that the ratios of the radii must be the same (up to inversion) in both pairs. (Prove all your claims carefully!)

5. Find a Möbius transformation which carries the circle $|z| = 1$ and the line $x = 2$ into concentric circles. What is the ratio of the radii?

Note: The home assignment 1 is to be left at latest on Friday, February 28th, at 5 p.m. in my mail-box.