EE128 FINAL EXAMINATION

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This document contains **seven** *pages.*

Problem 1.(15 %)
Using the root locus, find the range of gain for which the following system is unstable:

The plant has transfer function

$$G(s) = \frac{s+2}{s(s-2)(s^2+2s+10)} .$$

Problem 2.(15 %)

Consider the following linear system:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

1. Can you stabilize the system using output feedback? If yes, find the feedback law that places the closed-loop poles at $-1 \pm 2i$.

2. Can you stabilize the system using state feedback? If yes, find the feedback law that places the closed-loop poles at $-0.5 \pm 1.32i$.

Problem 3.(15 %)
Is the following system completely controllable?

$$\dot{x} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

If yes, find the change of variables so that the system is in controllable canonical form.

Problem 4.(15 %)
Let
$$G(s) = \frac{25(s+1)}{s(s+2)(s^2+2s+16)}$$
in the following system:

1. Draw the Bode diagrams.
2. Draw the Nyquist plot.
3. For which values of K is the system stable?
4. Sketch the root locus to verify your conclusion.

Problem 5.(10 %)
Consider the following ODE:

$$\begin{aligned}\dot{x}_1 &= x_2 + x_1 x_2^2 \\ \dot{x}_2 &= -x_1 + u(x_1, x_2)\end{aligned}$$

where $u(x_1, x_2)$ is a function of x_1 and x_2 (the control law) such that $u(0,0) = 0$. First observe that the point $(0,0)$ is an equilibrium point. Next design a control law (i.e., find a function $u(x_1, x_2)$) that will make the system asymptotically stable around the point $(0,0)$. *Hint:* Consider the function $V(x_1, x_2) = \frac{1}{2}(x_1^2 + x_2^2)$. Find $u(x_1, x_2)$ so that $V(x_1, x_2)$ is a Lyapunov function. Try a polynomial in x_1, x_2.

Problem 6.(15 %)

Consider the following discrete time system:

$$x(n+1) = \begin{bmatrix} 4 & 12 \\ -4.5 & -11 \end{bmatrix} x(n) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(n)$$

$$y(n) = \begin{bmatrix} 2 & -5 \end{bmatrix} x(n)$$

1. Find the transfer function from u to y.

2. Is the system BIBO stable?

Problem 7.(10 %)

Indicate whether True (T) or False (F) for each of the following questions. *Warning:* A correct response will give you 1 point, no response will give you no points, and a wrong response will take off 1 point, so that if you answer the questions at random, the average score will be 0 (as if you had omitted this problem completely).

1. The system $\dot{x} = x^2 + 5x$ has only one stable equilibrium point.

T **F**

2. In a unity feedback system the closed loop poles are continuous functions of the gain K.

T **F**

3. The solution of the linear system $\cdot x = A(t)x$, where $A(t)$ is a time-dependent matrix, is $x(t) = e^{A(t)t}$.

T **F**

4. When we discretize a stable continuous time system using forward differences the resulting discrete time system is always stable.

T **F**