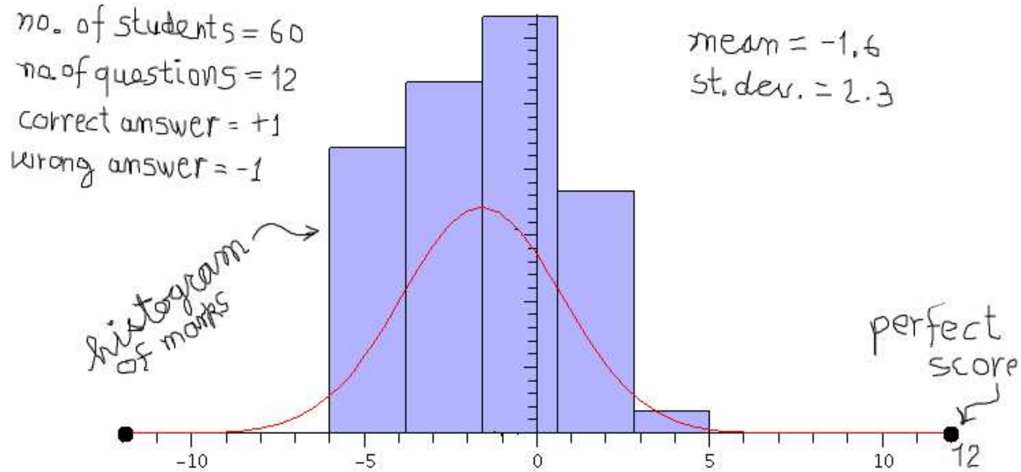
Results and samples of remarkable responses for the impromptu test for "Statistics 4" students Autumn 2006

This was a very elementary test.
Time allocated for the test: 45 minutes.
Number of students: 60.
Number of questions: 12.
Each answer was given +1 point if it was correct, and −1 point if it was totally wrong. No points were given for questions that were not attempted.
Mean score: −1.6.
Standard deviation: 2.3.
Histogram:



Here is a histogram across questions:

Fraction of correct answers to question 1:	2/60
Fraction of correct answers to question 2:	19/60
Fraction of correct answers to question 3:	1/60
Fraction of correct answers to question 4:	0
Fraction of correct answers to question 5:	1/60
Fraction of correct answers to question 6:	0
Fraction of correct answers to question 7:	21/60
Fraction of correct answers to question 8:	4/60
Fraction of correct answers to question 9:	49/60
Fraction of correct answers to question 10:	4/60
Fraction of correct answers to question 11:	38/60
Fraction of correct answers to question 12:	37/60

Some of the responses were totally surprising/remarkable. Below are some of them. In particular, I noticed that:

1. Only one out of 60 students could define what a derivative is.
2. No student could define what an integral is, or, at least, describe the concept verbally.
3. None out of the 60 students could give the correct negation of the statement "from tomorrow onwards, it will rain every day in Edinburgh".
4. I was suprised that elementary arithmetic problems, things that they should know at the age of 12, are alien to them.
5. Many could identify that 100^{200} is larger than 200^{100} but could not explain why.
6. Some did not know how to multiply two polynomials.
7. One conclusion is that they can work mechanically (a number of them did answer questions 11 and 12) but have no clue what they are calculating or why. a university.
8. Another conclusion is that it is not Mathematics education they lack, but elementary logic, and also ways to express themselves. (See, e.g. the responses for 3 and 4.)

SEMANTICS

1. Given two polynomials $p(x) = \sum_{k=0}^{n} a_k x^k$ and $q(x) = \sum_{k=0}^{m} b_k x^k$, express the coefficient of the term x^k of the product $r(x) = p(x)q(x)$ in terms of the coefficients (a_k) and (b_k).

    - $a_k + b_k$.
    - $a_k b_k$.
    - $a_{\sqrt{k}} b_{\sqrt{k}}$.
    - $a_{k^{1/2}} b_{k^{1/2}}$.
    - $\sum_{k=0} a_n b_m$.
    - $\binom{nm}{k} a_k b_k$.
    - $a_0 b_k + a_k b_0$.
    - $p(x)q(x) = \sqrt{\sum_{k=0}^{mn} a_k b_k}$
    - $p(x)q(x) = a_0 b_0 + a_1 b_1 x^2 + a_2 b_2 x^4 + \cdots a_{n-1} b_{n-1} x^{n^2-2n+1} + a_n b_n x^{2n}$
    - $p(x)q(x) = a_0 b_0 x^0 + a_1 b_1 x^2 + a_2 b_2 x^4 + \cdots + a_n b_m x^{m+n} = \sum_{k=0}^{m+n} a_k b_k x^{2k}$.
    - $p(x)q(x) = a_k b_k x^{2k}$.
    - $p(x)q(x) = \sum_{k=0}^{n} \sum_{k=0}^{b} a_k b_k x^{k^2}$.
    - $p(x)q(x)$= [an expression that does not even contain x].
    - $p(x)q(x) = x^k (\sum_{k=0}^{n} a_k \sum_{k=0}^{m} b_k)$.
    - $p(x)q(x) = (\sum_{k=0}^{n+m} a_k b_k) x^k$.
    - $p(x)q(x) = \sum_{k=0}^{m+n} a_k b_k x^k = \frac{p(x)q(x)}{x^k} = \sum_{k=0}^{m+n} a_k b_k$.

2. Simplify the expression

$$\prod_{k=1}^{N-1} \left(\frac{k}{k+1}\right)^2$$

    - $(\frac{1}{1+1})^2 + (\frac{2}{3})^2 + \cdots + \frac{n-1}{n}$.
    - $(\frac{1}{2})^2 + (\frac{2}{3})^2 + (\frac{3}{4})^2 + \cdots + (\frac{n}{n+1})^2$.
    - $\prod_{k=1}^{N-1} -1$.
    - $1 - \prod_{k=1}^{N-1} \frac{2}{k+1} + \prod_{k=1}^{N-1} \frac{1}{(k+1)^2}$.
    - $\prod_{k=1}^{N-1} (\frac{k}{k+1})^2 = \frac{1}{2/k+1/k^2}$.
    - $\prod_{k=1}^{N-1} (\frac{k}{k+1})^2 = \frac{1}{4 \times 9 \times 16 \times \cdots \times N^2}$.

CONCEPTS

3. Define the concept of the derivative of a function $f : \mathbb{R} \to \mathbb{R}$ at a point $x \in \mathbb{R}$.

    - `Derivative is the function multiplied by its power and the power decreased by one. (Dont know otherwise.) its a real number, when put through the function` n `produces another real number.`
    - `derivative of` f `at` $x \in \mathbb{R}$ `limit of` $f(x)$ `as at` x `exist` $\lim_{\texttt{as}x \to 0} f(x)$ `exists.`
    - $\frac{df}{dx} = \frac{f(x)-\Delta x}{\Delta x}$ `as` $\Delta x \to 0$.
    - `The derivative of a function` $f : \mathbb{R} \to \mathbb{R}$ `at a point` $x \in \mathbb{R}$ `is the value that a function takes at a particular point` x.
    - `This is the distance of the point` x `from the origin on a plain.`
    - `A real number` x `is mapped to another real number by the function` f. `The derivative of this number is the rate of change of` x `to the new real number.`
    - `The derivative of the function is the gradient of the function at a point` x. `It calculates the 'slope' of the 'curve' at that point for` x,
    - `The derivative of a function` $f : \mathbb{R} \to \mathbb{R}$ `is the gradient of the graph of the function at the point` x.
    - $f'(x)$ `=differentiate the function` $f(x)$.

- the derivative of function f is donated [sic] f' and is defined $\frac{df(x)}{dx}$ at point $x, (x \in \mathbb{R})$.
- You are finding the derivative of function from the real number which when you've found the derivative will be in the real nubers if x is a real number [sic].
- Rate of change of a function f at point x.
- We are trying to differentiate a[n] equation so that we can find a simplier [sic] form of the equation. It help[s] in finding tangents for curves.
- The variation of f at a point x.
- The derivative of a function f which consists of a point, in this case the real point x, specifies the rate at which the function f chamges with respect to a change in x. This is because f is defined by the independent variable x and would theorefore depend on it.
- Any function involving x as a real number will differentiate to a real number.
- The derivative of a function of a point $x \in \mathbb{R}$ is the value of that variable with respect to another variable, i.e. y.
- $f : \mathbb{R} \to \mathbb{R}$ at a point $x \in \mathbb{R}$ is just deriving a point that is a real number and when it has been derived it is still a real number.
- The derivative at a point x is what you get back out when you put point x into the derivative. It will also be a real number.
- A function f will be differentiated. The input will be a values $\in \mathbb{R}$, $x \in \mathbb{R}$, the function will then give an output which also $\in \mathbb{R}$.

4. Explain what we mean by the integral $\int_0^1 f(x)\ dx$ of a function $f : [0, 1] \to \mathbb{R}$. (The answer "area under the curve" is not acceptable.)

- $\int_0^1 f(x)\ dx$ is the segment of an area contained within the function $f(x)$, the x-axis and it is defined by the two limits 1 and 0 on the x axis.
- $\int_0^1 f(x)\ dx\ f : \mathbb{R} \to \mathbb{R}$ at a point $x \in \mathbb{R}$ is the integral of the function of x which encloses the area between the curve and between the intervals $x = 1$ and $x = 0$. x takes values $\in \mathbb{R}$ and the function maps this, which is also a subset of the real numbers.
- $\int_0^1 f(x)\ dx$, $f : \mathbb{R} \to \mathbb{R}$ = integrate $f(x)$ w.r.t. x over the range of real no. which lies between 0 and 1.
- $\int_0^1 f(x)\ dx$ is the area under curve $f(x)$ from $x = 0$ to $x = 1$ it's [plot of function] $\delta x \to 0$.
- By integrating this function, we are being asked to calculate an area, and by providing definate [sic] integrals, the question asks us to provide a specific area.
- This is what is known as a definite integral. It is used to give areas of graphs e.g. [plot follows].
- The area under the curve between 0 and 1 given in units^2.
- $\int_0^1 f(x)\ dx = [\frac{d}{dx} f(x)]_{x=0}^1 = \frac{d}{dx} f(1) - \frac{d}{dx} f(0)$
- $\int_0^1 f(x)\ dx$ a number from the domain $[0, 1]$ is inputed into the function and will give an output belonging to the codomain which is the set of $\mathbb{R}$ numbers.
- It means to get the area under the curve $f(x)$ from the point $x = 0$ to $x = 1$. It is by integrating $f(x)$ w.r.t. x.
- The inverse of the derivative of a function.
- $\int_0^1 f(x)\ dx = sum_{x \in [0,1]} f(x) dx$ sum the heights of $f(x)$ multiplied by the distance between the heights for all x in the interval.
- $\int_0^1 f(x)\ dx$ of a function $f : [0, 1] \to \mathbb{R}$ Using the function $f(x)$ when integrated between the limit $[0, 1]$ a real number value will be resulted.
- The integral $\int_0^1 f(x)\ dx$ of a function $f : [0, 1] \to \mathbb{R}$ is the value of the space between the interval 0, 1, and the x-axis.

- $\int_0^1 f(x)\ dx$ of a fn $f : [0,1] \to \mathbb{R}$ is the value of the interval between $[0,1]$ and the x-axis to $\mathbb{R}$.
- The sum of the values that the function takes between the limits of 0 and 1.
- The integral $\int_0^1 f(x)\ dx$ takes the values of the variable x between zero and one, puts them into the function $f(x)$ and creates an image of these values of x in $\mathbb{R}$. We can then plot these images on a graph.
- The sum of all points $f(x)$ can assume under the range of $0 \leq x \leq 1$.
- It is the value of the function in the interval $[0,1]$. This value output is a real number.

LOGIC AND COUNTING

5. Using the word "dry day", write a sentence which expresses the negation of the clause below: *"From tomorrow onwards, it will rain every day in Edinburgh."* (A dry day is a day during which it never rains.)

   - Until yesterday downwards, it won't dry any day outside Edinburgh.
   - From today backwards, it was a dry day every day in Edinburgh.
   - From tomorrow onwards, every day will be a dry day in Edinburgh.
   - From tomorrow onwards, it will be a dry day every day in Edinburgh.
   - From tomorrow onwards, there will be no more dry days in edinburgh.
   - From tomorrow onwards, there will only be dry days in Edinburgh.
   - From tomorrow onwards, the probability of ''dry day'' is zero.
   - From tomorrow onwards, it is unlikely that a dry day will occur in Edinburgh.
   - A dry day will never occur in Edinburgh from tomorrow onwards.
   - As of today, there will not be one dry day.
   - There will be no dry day beyond tomorrow.
   - From tomorrow onwards, it will be dry day every day in Edinburgh.
   - As of tomorrow, there will be a dry day every day onwards in Edinburgh.
   - The number of ''dry days'' in Edinburgh will equate to zero.
   - It will not be a dry day from tomorrow onwards.
     (dry day)^c $= 1-$(wet days).
   - Today is a dry day but as of tomorrow there won't be a dry day.
   - Today will be the last ever dry day in Edinburgh.

6. In how many ways can you put 5 indistinguishable balls in 7 distinctly numbered boxes and why?

   - 5^7.
   - $\binom{7}{5}$.
   - $7 + 2 \times 6^7 + C_7^5 + C_7^4 \times 4 + C_7^1 \times 6$.
   - 119.
   - 21/5.

ARITHMETIC

7. Find the largest of the two integers 100^{200}, 200^{100} (and explain why).

   - $100^{200} = 200^{100}$ -- same number of zeros involved and the two's involved.
   - 100^{200} is larger, just seems it, 100 is a big number, only half the size of 200 and is getting multiplied by itself 2wice many times can't explain
   - 100^{200} is bigger because although it is 20 to the power 1000, it is just adding 1000 zeros behind 20 while for the other one, we have 2000 zeros.
   - 100^{200} is the largest number because it has a bigger order.

- $100^{200} = \sum_{n=1}^{200} 100^n$.
- $100^{200} = 10^4 00$, $200^{100} = 20^{400}$. $100^{200} < 200^{100}$ as they are now expressed in terms of the same power implying that the greater outcome will belong to that with a similar base in this case 20.
- 100^{200} is bigger as it is being multiplied 100 times more than the other number.

8. What is the least common multiple of 140 and 650? What is their greatest common divisor?

- least common multiple is 3
- least common multiple is 1
- least common multiple is 10
- least common multiple: 1820
  greatest common divisor: 910
- 2= least common multiple

ALGEBRA

9. Expand $(a+b)^5$, where a, b are real numbers.

- $(a+b)^5 = \binom{5}{0}a^5 + \binom{4}{1}a^4b + \binom{3}{2}a^3b^2 + \binom{2}{3}a^2b^3 + \binom{1}{4}ab^4 + b^5$ binomial expansion
- $(a+b)^5 = a^5 + 3a^4b + 9a^3b^2 + 11a^2b^3 + 7ab^4 + b^5$.
- $(a+b)^5 = \binom{a}{0}a^5 + \binom{a}{1}a^4b + \binom{a}{2}\frac{a^3b^2}{2!} + \binom{a}{3}\frac{a^2b^3}{3!} + \binom{a}{4}\frac{ab^4}{4!} + \binom{a}{5}\frac{b^5}{5!}$.
- $(a+b)^5 = a^5 + a^4b + a^3b^2 + a^2b^3 + ab^4 + b^5$.

10. Express $1 + x^2 + x^4 + \cdots + x^{20}$ as a ratio of two polynomials.

- $1 + x^2 + x^4 + \cdots + x^{20} = (1+x^2)^{10}$.
- $1 + x^2 + x^4 + \cdots + x^{20} = \frac{\sum_{k=0}^{20} x^k}{\sum_{k=0}^{19} x^{2k+1}}$.
- $\frac{x^{20}}{(1-x^2)!}$.

(BRUTE FORCE) CALCULUS

11. What is the derivative of the function $f(x) := x\cos(x^2)$?

- $\cos^2 x - 2x^2 \sin x^2$.
- $\cos(x^2) + x\cos(x^2) + x(2\sin x \cos x)$.
- $\cos(x^2) - 2x^3 sin(x^2)$.
- $\frac{1}{2}\sin(x^2) + C$.
- $\frac{-1}{2}\sin x^2$.
- $\frac{1}{2x}\cos x^2 + x\sin x^2$.

12. Compute the (indefinite) integral $\int \frac{1}{\sqrt{x}}\, dx$.

- $-\frac{2}{x^3} + C$.
- $\frac{1}{2\sqrt{x}} + C$.
- $-\frac{2}{3(x)^{3/2}} + C$.
- $\ln x^{1/2} + C$.
- $-2u + C$.