Misprints and Corrections to

An Intermediate Course in Probability

Misprints

Page	*Line*	*Text*	*Should have been*
25	15	$f_{u,v}(u,v) = 0$	$f_{U,V}(u,v) = 0$
27	5	diameter	radius
27	11	domain	range
49	14	The function $h(\mathbf{x})$	The function h
51	9	for $\quad x, y > 0,\ 0 < x, y < 1$	for $\quad 0 < x, y < 1$
62	13	$\|t\| < 1$	$\|t\| \leq 1$
65	9	Corollary 2.2.1	Theorem 2.2
65	18	$t < \frac{1}{q}$	$\|t\| < \frac{1}{q}$
69	1	Let $X_n \in \text{Bin}(n, p)$.	Let $X \in \text{Bin}(n, p)$.
97	3_-	persons	passengers
124	3	independent components	independent normal components
128	14	singular	nonsingular
140	8_-	*(*Rank $Q_i =$)	(Rank $Q_i =$)
141	4	$n = 2$.	$k = 2$.
141	6	$n = 2$:	$k = 2$:
141	9	By assumption,	Since A_1 is nonnegative definite,
141	4_-	$n = 2$.	$k = 2$.
151	5	Section 5	Section 6
156	8	Let $X_1,\ X_2,\ \ldots$	Let $X_2, X_3, \ldots$
156	9	$n \geq 1$	$n \geq 2$
162	6	in definition	in the definition
183	12	$g(X(\omega))$	$g(X_n(\omega))$
183	12	$X(\omega)$	$X_n(\omega)$
183	14	$g(X)$	$g(X_n)$
183	14	X	X_n
184	8	$\sum_{k=1}^n X_i^2$	$\sum_{k=1}^n X_k^2$
184	10	$\sum_{k=1}^n X_i^2$	$\sum_{k=1}^n X_k^2$

184	13	$\sum_{k=1}^{n} X_i^2$	$\sum_{k=1}^{n} X_k^2$
187	14	$Y_n = \min\{X_1, X_2, \dots, X_n\}$	$Y_n = \max\{X_1, X_2, \dots, X_n\}$
188	11	and $n = 1, 2, \dots$.	and $n = 2, 3, \dots$.
196	5	$\leq < t_{n-1}$	$\leq t_{n-1}$
200	7_-	$g(t)$	$g(t, s)$
200	5_-	Po	Po
227	17_-	$T_y \in$ Exp$(\frac{1}{\lambda})$.	$T_y \in$ Exp$(\frac{1}{\lambda})$, that is, $E\,T_y = \frac{1}{\lambda}$.
235	15_-	picture)such	picture) such
247	1_-	$X_1(0) = 9$	$X_1(3) = 9$
248	1	$X_2(0) = 5$	$X_2(3) = 5$
266	1_-	$x > 0\ n > 2\ n > 2$	$x > 0\ n > 2\ n > 4$
267	16	≈ 0.1006	≈ 0.1003
268	5	27. $a = \frac{12}{7}$ $\quad b = \frac{3}{14}$.	27. $a = b = 3/7$.

Corrections

Page 38: In Theorem 2.3 it must also be assumed that $E\,Y^2 < \infty$ and that $E(g(X)) < \infty$.

Page 72, line 13: Replace this line by the following: $\log t + \mu + \frac{1}{2}\sigma^2 n \geq \frac{1}{4}\sigma^2 n$ for any fixed $t > 0$ as $n \to \infty$ and $\exp\{cn^2\}/n! \to \infty$

Page 161, Theorem 3.3: We also assume that $E|X_n|^r < \infty$ for all n.

Page 184, Example 7.7: It is not necessary that V_n and Z_n are independent for the conclusion to hold. (It is, however, necessary in order for T_n to be t-distributed, which is of statistical importance; cf. Remark 7.3, page 185.)

Pages 203-204: Replace the piece following formula (1.13) until Remark 1.2 by the following:

This proves (a) for the case $k = 2$. In the general case (a) follows similarly, but the computations become more (and more) involved. We carry out the details for $k = 3$ below, and indicate the proof for the general case. Once (a) has been established (b) is immediate.

Thus, let $k = 3$ and $0 \leq s \leq t \leq u$. By arguing as above, we have

$$\begin{aligned}
&P(T_1 \leq s < T_2 \leq t, T_3 > u) \\
&\qquad = P(X(s) = 1, X(t) = 2, X(u) < 3) \\
&\qquad = P(X(s) = 1, X(t) - X(s) = 1, X(u) - X(t) = 0) \\
&\qquad = P(X(s) = 1) \cdot P(X(t) - X(s) = 1) \cdot P(X(u) - X(t) = 0) \\
&\qquad = \lambda s e^{-\lambda s} \cdot \lambda(t-s)e^{-\lambda(t-s)} \cdot e^{-\lambda(u-t)} = \lambda^2 s(t-s)e^{-\lambda u},
\end{aligned}$$

and

$$\begin{aligned}
&P(T_1 \le s < T_2 \le t, T_3 \le u) + P(T_1 \le s < T_2 \le t, T_3 > u) \\
&\quad = P(T_1 \le s < T_2 \le t) = P(X(s) = 1, X(t) \ge 2) \\
&\quad = P(X(s) = 1, X(t) - X(s) \ge 1) \\
&\quad = P(X(s) = 1) \cdot (1 - P(X(t) - X(s) = 0)) \\
&\quad = \lambda s e^{-\lambda s} \cdot (1 - e^{-\lambda(t-s)}) = \lambda s(e^{-\lambda s} - e^{-\lambda t}).
\end{aligned}$$

Next we note that

$$\begin{aligned}
F_{T_1,T_2,T_3}(s,t,u) &= P(T_1 \le s, T_2 \le t, T_3 \le u) \\
&= P(T_2 \le s, T_3 \le u) + P(T_1 \le s < T_2 \le t, T_3 \le u),
\end{aligned}$$

that

$$\begin{aligned}
&P(T_2 \le s, T_3 \le u) + P(T_2 \le s, T_3 > u) \\
&\quad = P(T_2 \le s) = P(X(s) \ge 2) = 1 - P(X(s) \le 1) \\
&\quad = 1 - e^{-\lambda s} - \lambda s e^{-\lambda s},
\end{aligned}$$

and that

$$\begin{aligned}
P(T_2 \le s, T_3 > u) &= P(X(s) \ge 2, X(u) < 3) \\
&= P(X(s) = 2, X(u) - X(s) = 0) \\
&= P(X(s) = 2) \cdot P(X(u) - X(s) = 0) \\
&= \frac{(\lambda s)^2}{2} e^{-\lambda s} \cdot e^{-\lambda(u-s)} = \frac{(\lambda s)^2}{2} e^{-\lambda u}.
\end{aligned}$$

We finally combine the above to obtain

$$\begin{aligned}
F_{T_1,T_2,T_3}(s,t,u) &= P(T_2 \le s) - P(T_2 \le s, T_3 > u) \\
&\quad + P(T_1 \le s < T_2 \le t) - P(T_1 \le s < T_2 \le t, T_3 > u) \\
&= 1 - e^{-\lambda s} - \lambda s e^{-\lambda s} - \frac{(\lambda s)^2}{2} e^{-\lambda u} \\
&\quad + \lambda s(e^{-\lambda s} - e^{-\lambda t}) - \lambda^2 s(t-s) e^{-\lambda u} \\
&= 1 - e^{-\lambda s} - \lambda s e^{-\lambda t} - \lambda^2 (st - \frac{s^2}{2}) e^{-\lambda u}, \qquad (1.14a)
\end{aligned}$$

and, after differentiation,

$$f_{T_1,T_2,T_3}(s,t,u) = \lambda^3 e^{-\lambda u}, \quad \text{for} \quad 0 < s < t < u. \tag{1.14b}$$

The change of variables $\tau_1 = T_1$, $\tau_1 + \tau_2 = T_2$, and $\tau_1 + \tau_2 + \tau_3 = T_3$ concludes the derivation, yielding

$$f_{\tau_1,\tau_2,\tau_3}(v_1,v_2,v_3) = \lambda e^{-\lambda v_1} \cdot \lambda e^{-\lambda v_2} \cdot \lambda e^{-\lambda v_3}, \tag{1.14c}$$

for $v_1, v_2, v_3 > 0$, which is the desired conclusion.

Before we proceed to the general case we make the crucial observation that the probability $P(T_1 \le s < T_2 \le t, T_3 > u)$ was the only quantity containing *all* of s, t, and u and, hence, since differentiation is with respect to *all* variables, the only one that contributed to the density. This carries over to the general case, that is, it suffices to actually compute only the probability containing all variables.

Thus, let $k \ge 3$ and let $0 \le t_1 \le t_2 \le \ldots \le t_k$. In analogy with the above we find that the crucial probability is precisely the one in which the T_i are separated by the t_i. It follows that

$$\begin{aligned} F_{T_1,T_2,\ldots,T_k}&(t_1,t_2,\ldots,t_k) \\ &= -P(T_1 \le t_1 < T_2 \le t_2 < \ldots < T_{k-1} \le t_{k-1}, T_k > t_k) \\ &\qquad + R(t_1,t_2,\ldots,t_k) \\ &= -\lambda^{k-1} t_1(t_2-t_1)(t_3-t_2)\cdots(t_{k-1}-t_{k-2})e^{-\lambda t_k}, \end{aligned} \tag{1.15a}$$

where $R(t_1, t_2, \ldots, t_k)$ is a remainder containing the probabilities of lower order, that is, those for which at least one t_i is missing.

Differentiation now yields

$$f_{T_1,T_2,\ldots,T_k}(t_1,t_2,\ldots,t_k) = \lambda^k e^{-\lambda t_k}, \tag{1.15b}$$

which, after the transformation $\tau_1 = T_1$, $\tau_2 = T_2 - T_1$, $\tau_3 = T_3 - T_2$, $\ldots$, $\tau_k = T_k - T_{k-1}$, shows that

$$f_{\tau_1,\tau_2,\ldots,\tau_k}(u_1,u_2,\ldots,u_k) = \prod_{i=1}^{k} \lambda e^{-\lambda u_i}, \tag{1.15c}$$

for $u_1, u_2, \ldots, u_k > 0$, and we are done. □

Page 207: Formula (1.21) only works for (and, hence, (1.22) has only been strictly demonstrated for) $j > 1$. The following modifications show that (1.22) holds for $j = 0, 1$ (actually, these cases are easier):

Let $i = 0$ and $j = 0$. We have

$$\begin{aligned} P(X(s) = 0,\, X(s+t) - X(s) = 0) &= P(X(s+t) = 0) \\ &= P(T_1 > s+t) = e^{-\lambda(s+t)} \\ &= e^{-\lambda s} \cdot e^{-\lambda t}\,, \end{aligned}$$

which is (1.22) for that case.

For $i = 0$ and $j = 1$ we have

$$\begin{aligned} P(X(s) = 0,\, X(s+t) - X(s) = 1) &= P(X(s) = 0,\, X(s+t) = 1) \\ &= P(s < T_1 \le s+t < T_2) \\ &= \int_{s+t}^{\infty} \int_{s}^{s+t} f_{T_1,T_2}(t_1, t_2)\, dt_1 dt_2. \end{aligned}$$

Inserting the expression for the density as given by (1.20) (with $k = 1$) and integration yields

$$P(X(s) = 0,\, X(s+t) - X(s) = 1) = e^{-\lambda s} \cdot \lambda t e^{-\lambda t}\,,$$

which is (1.22) for that case.

Page 209-210, Example 2.1.(b): The solution should be replaced by

(b) Let $\tau_1, \tau_2, \ldots$ be the times between cars. Then $\tau_1, \tau_2, \ldots$ are independent, $\mathrm{Exp}(\frac{1}{15})$-distributed random variables. The actual waiting times, however, are $\tau_k^* = \tau_k \mid \tau_k \le 0.1$, for $k \ge 1$. Since there are N cars passing before she can cross, we obtain

$$T = \tau_1^* + \tau_2^* + \ldots + \tau_N^*,$$

which equals zero when N equals zero. It follows from Section III.5 that

$$E\,T = E\,N \cdot E\,\tau_1^* = (e^{1.5} - 1) \cdot \Big(\frac{1}{15} - \frac{0.1}{e^{1.5} - 1}\Big) = \frac{e^{1.5} - 2.5}{15}. \qquad \square$$

Uppsala, 20 August, 2008 Allan Gut