# Probability: A Graduate Course, Corr. 2nd printing

Misprints and Corrections

13 mars 2012

## 1. Misprints

| Page | Line/Problem | Should be |
|---|---|---|
| 14 | L11 | $\{A_{k,\varepsilon} \in \mathcal{F}_0,\ 1 \le k \le n_*\}$ |
| 35 | L6$_-$ | Lemma 2.1: |
| 37 | L11 | For the proof of the theorem |
| 37 | L12 | proof, in turn, of these facts we refer |
| 42 | L2-3 | bution function on $C_n$ defined by $F_n(0) = 0$, ... |
| 42 | L10 | $F(x) - F(y)$ |
| 42 | L12 | $F'(x) = 0$ for all $x \notin C$. |
| 51 | L20 | $\varepsilon < c$ |
| 56 | L6 | Since $0 \le X_1 - X_n \nearrow X_1 - X$ |
| 57 | L3$_-$ | converges a.s. (delete as $n \to \infty$) |
| 61 | L5 | $\sum_{k=1}^n x_k P(A_k)$ |
| 77 | L12 | $\int_0^\infty \cdots\, dx = \frac{1}{\log \lambda} \int_0^\infty \cdots\, dy$ |
| 77 | L1$_-$ | Theorem 12.3. |
| 78 | L11$_-$ | such that $n_0 = 0$ and $n_{k+1} \ge \lambda n_k$ |
| 78 | L9$_-$ | $\le \sum_{k=1}^\infty \lambda^{-1} n_k P(X \ge n_k) +$ |
| 98 | L9 | following result |
| 98 | L14 | The corollary therefore |
| | | |
| 106 | L11 | $= 1$ (i.e. not $\to 1$) |
| 108 | L10 | Theorem 18.7 |
| 108 | L6$_-$ | Since $P(A_n) \to 0$ as $n \to \infty$, Theorem 18.7 |
| 133 | L12$_-$, 5$_-$ | $P(\cdots) \le 1/2$ |
| 135 | L4 | $C_k = \{X_k^s \ge x\}$ |
| 135 | L13 | $P\Big(\bigcup_{k=1}^n C_k\Big) \ge P(\bigcup_{k=1}^n \{A_k \cap B_k\}) = \sum_{k=1}^n P(A_k \cap B_k) =$ |
| 142 | L12$_-$ | $\lvert S_n\rvert \le \cdots$ (not $\lvert S_k\rvert \le \cdots$) |
| 145 | L12 | $P(\lvert S_n\rvert > x) = \sum_{k=1}^n P(\cdots) + P(\{\lvert S_n\rvert > x\} \cap A_n^c)$ |
| 150 | L17 | $T_n^s(t) = \sum_{k=1}^n X_k^s r_k(t)$. |
| 151 | L8 | $B_p n^{p/2} E\lvert X\rvert^p$ |
| 152 | L10$_-$ | Set $Q_n^*(X) = (\sum_{k=1}^n E\, X_k^2)^{1/2}$. |
| 152 | L3$_-$, 2$_-$ | $\max\{..., Q_n^*(X)\}$ |
| 166 | L3$_-$ | Section 2.11 (i.e. not Subsection 2.2.6) |
| 168 | L6$_-$ | Section 2.11 (i.e. not Subsection 2.2.6) |
| 172 | L5 | members with $e^{-iuy}$ (i.e. not $e^{-ity}$) |
| 177 | L3$_-$ | $n = 2$ |
| 185 | L15 | $\kappa_X(t) = \sum_{k=1}^n \cdots$ (i.e. delete 1+) |
| 186 | L7 | $\gamma_1 = \frac{\varkappa_3}{\varkappa_2^{3/2}}$ (not $\varkappa_1^{3/2}$) |

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| Page | Line/Problem | Should be |
|---|---|---|
| 190 | $L7_-$ | for (i.e. not fot) |
| 196 | L2 | $\sim C\big(\cdots\big)^{(\beta+k)/\alpha}/\sqrt{\cdots}$ (divided by $\sqrt{\cdots}$) |
| 196 | L4 | $=\sum_{k=1}^{\infty}\big(\cdots\big)^{\cdots}\big(\frac{\beta+k}{\alpha}\big)^{1/4k}$ |
| 199 | $L6_-$ | (Section 2.15) |
| | | |
| 202 | $L10_-$ | (or mean-square (i.e. delete to the) |
| 217 | L8 | integrable random variables. |
| 218 | $L5_-$ | Let $r>0$, and suppose that $E\lvert X_n\rvert^r<\infty$ for all $n$. |
| 219 | $L3_-$ | The third term converges to 0 as $n\to\infty$ by Theorem |
| 221 | L2 | Let $r>0$, and suppose that $E\lvert X_n\rvert^r<\infty$ for all $n$. |
| 224 | L4 | Proposition 2.2.1(c) (i.e. not Theorem 2.2.1(iii)) |
| 224 | L12 | Let $r>0$, and suppose that $E\lvert X_n\rvert^r<\infty$ for all $n$. |
| 229 | $L10_-$ | $n\geq 2$ (not $n\geq 1$) |
| 236 | L3 | Proposition 2.2.1 (i.e. not Lemma 2.2.1) |
| 257 | $L8_-$ | $X_m-X_{n_k}\overset{a.s.}{\to}X_m-X$ as $k\to\infty$. |
| 262 | P12, L4 | $n=2,3,\ldots$. |
| 262 | P12, L5 | $n\geq 2$ |
| 271 | $L1_-$ | Theorem 5.3.3 |
| 285 | L6 | where here and in the following, $S_n=\sum_{k=1}^{n}X_k$, $n\geq 1$, then this |
| 285 | L18 | Then (delete "with partial sums $S_n$, $n\geq 1$") |
| 286 | L18 | random variables. (delete "with partial sums $S_n$, $n\geq 1$") |
| 293 | L3 | $\prod_{k=1}^{n}\varphi_{X_k^s}(t)=\prod_{k=1}^{n}\lvert\varphi_{X_k}(t)\rvert^2\to\lvert\varphi(t)\rvert^2$ as $n\to\infty$, |
| 297 | L4 | $\frac{n-1}{n}\cdot\frac{S_{n-1}}{n-1}$ (not $\frac{n-1}{n}\cdot\frac{S_n}{n}$) |
| | | |
| 325 | $L1_-$ | $n\geq 2$ (not $n\geq 1$) |
| 337 | $L3_-$ | delete $\to 0$ |
| 338 | $L2_-$ | Theorem 4.4.1 |
| 346 | L1 | ... given $\epsilon>0$ and $\eta>0$, there exist $\delta>0$ and $n_0$, .... |
| 355 | L15 | $\gamma^3=E\lvert X-\mu\rvert^3$ (not just $E\lvert X\rvert^3$) |
| 357 | L5 | ... with mean 0 and partial |
| 375 | L8 | $+\frac{t-(k-1)/n}{1/n}\frac{\xi_k(\omega)}{\sigma\sqrt{n}}$ |
| 380 | P23, L9 | In terms of $L^p$-norms (not Euclidean) |
| 386 | L4 | apply Lemma 3.1.1 (not Markov's inequality) |
| 386 | $L9_-, 8_-$ | application of Lemma 3.1.1 (not Markov's inequality) |
| 392 | $L8_-, 6_-$ | $(E\,X'_{1,n})^2$ (not $(E\,X_{1,n})^2$ ) |
| 392 | $L6_-$ | $E\,(X'_{1,n})^2$ (not $E\,X_{1,n}^2$ ) |
| 393 | $L7_-$ | delete almost surely |
| | | |
| 401 | L3,4 | $P(\lvert\cdots\rvert>\varepsilon\sqrt{\cdots})$ ($\varepsilon$ is missing) |
| 402 | L8 | since the two other |
| 403 | $L13_-8$ | From here on one can argue |
| 404 | L7 | dense, and, in Subsection 7.2, some that are not covered |
| 415 | L6 | visible, since the new |
| 420 | P4c | as $t\to\infty$ (not $n\to\infty$) |
| 425 | L $12_-$ | $=P(S_{n^k}>c_{n^k}x)\leq 2P(S_{(n+1)^k}>c_{n^k}x)$ (delete $c$ in the index) |
| 425 | L $5_-$ | established |
| 428 | L $11_-$ | (a) ... $V_n\overset{d}{\to}\frac{U-b}{a}$ |
| 429 | $L1_-$ | $j\to\infty$ |
| 430 | L6 | The proof of the theorem |

| | | |
|---|---|---|
| 452 | L3$_-$ | Fréchet: $\Phi_\alpha(x) = \begin{cases} 0, & \text{for } x \le 0, \\ \cdots, & \text{for } x > 0, \end{cases} \quad \cdots;$ |
| 452 | L2$_-$ | Weibull: $\Psi_\alpha(x) = \begin{cases} \cdots, & \text{for } x \le 0, \\ 1, & \text{for } x > 0, \end{cases} \quad \cdots;$ |
| 457 | L2 | $= -n(1 - F(a_n) \cdot \frac{1-F(a_n x)}{1-F(a_n)} \to$ |
| 457 | L5$_-$ | distributions for record values are |
| 459 | L6$_-$ | $\sum_{k=1}^{n} P(X_k \neq Y_k)$ (not $Y_n$) |
| 464 | P2,L1 | i.i.d. strictly stable |
| 464 | P8,L3 | $\frac{\log \lvert S_n \rvert}{\log n} \xrightarrow{p} \frac{1}{\alpha}$ as $n \to \infty$. |
| 466 | P8,L2 | canonical representation of Theorem 4.4 of the |
| 474 | L5$_-$ | $= E\{E\bigl(YE(Y \mid \mathcal{G}) \mid \mathcal{G}\bigr)\}$ (one $E($ too many) |
| 475 | L6 | of $\mathcal{F}$. If $g(X)$ is $\mathcal{G}$-measurable, then |
| 494 | L10$_-$ | the answer to (c) is negative |
| | | |
| 508 | L14 | Subsection 10.2 |
| 509 | L12 | Kolmogorov-Doob inequality |
| 516 | L14 | Kolmogorov-Doob inequality |
| 516 | L4$_-$ | Subsection 10.4 |
| 523 | L8$_-$ | $\xrightarrow{a.s.} \sum_{k=1}^{\tau} \lvert Y_k \rvert$ (not $\tau \wedge n$) |
| 525 | L4$_-$ | converges to $pE\,\tau$ |
| 526 | L12 | $E\,u^\tau = \frac{pu}{1-(1-p)u}$ |
| 530 | L9$_-$ | $\xrightarrow{a.s.} \frac{1}{E\,Y} \cdot E\,S_\tau$ |
| 532 | L14 | ment of the theorem. (not proposition) |
| 534 | L14$_-$ | $\{\lvert X_0 \rvert, \lvert X_{\tau \wedge n} \rvert, \lvert X_n \rvert\}$ |
| 539 | L11 | Then, for $u \in G = \{y : g(y) < \infty\}$, with $g'(u) \ge 0$, we have |
| 539 | L16 | We begin by proving a.s. convergence. |
| 539 | L17 | Suppose that $2u \in G$. Since $0 \in G$, it follows, via strict convexity, that |
| 539 | L10$_-$ | as $n \to \infty$. However, recalling that $2g(u) - g(2u)$, we conclude that |
| 539 | L9$_-$ | $X_\infty \xleftarrow{a.s.} \exp\{2uS_n - ng(2u))\} = \exp\{2(uS_n - ng(u))\}) \cdot \exp\{n\bigl(2g(u) < g(2u)\bigr)\}$ |
| 539 | L8$_-$ | $\xrightarrow{a.s.} X_\infty^2 \cdot 0$ as $n \to \infty$, |
| 539 | L7$_-$ | from which it follows that $X_\infty = 0$ a.s. |
| 540 | L10$_-$ | ... variable, and recalling that $g'(u) \ge 0$, |
| 540 | L8$_-$ | $= \exp\{-g(u)\}\psi'(u) = \frac{\psi'(u)}{\psi(u)} = g'(u) \ge 0.$ |
| 544 | L8 | (a) $\{\lvert X_n \rvert^p,\, n \le 0\}$ (not $n \ge 1$) |
| 544 | L8$_-$ | (a) $\{X_n,\, n \le 0\}$ (not $n \ge 1$) |
| 544 | L1$_-$ | For $n \ge 1$ (not $n \le -1$) |
| 564 | L5$_-$ | $\to a$ (not $\to 0$) |
| 565 | L 2 | $\limsup_{n\to\infty} \left\lvert \frac{1}{n} \sum_{k=1}^{n} \cdots \right\rvert$ (not $\sum_{k=1}^{n_0}$) |
| 565 | L 7 | $\frac{1}{\log\log n} \sum_{k=2}^{n} \cdots$ (not $\sum_{k=1}^{n}$) |
| 566 | L3 | Following is a continuous version. |
| 566 | L4 | real valued, uniformly bounded functions |
| 566 | L9 | prove a version |
| 566 | L8$_-$ | with a positive finite limit |

## 2. Corrections

- Page 106: Delete Theorem 18.9.

- Page 293: Lemma 4.4.1, the fact that $1 - x < -\log x$ for $0 < x < 1$, and (5.1), together show that

$$\begin{aligned}\sum_{n=1}^{\infty} P(|X_n^s| > 2/h) &\le \sum_{n=1}^{\infty} \frac{1}{h}\int_{|t|<h} (1 - |\varphi_{X_n}(t)|^2)\,dt \\ &= \frac{1}{h}\int_{|t|<h} \Big(\sum_{n=1}^{\infty}(1 - |\varphi_{X_n}(t)|^2)\Big)\,dt \le \frac{1}{h}\int_{|t|<h} -\sum_{n=1}^{\infty}\log\big(|\varphi_{X_n}(t)|^2\big)\,dt \\ &= \frac{1}{h}\int_{|t|<h} -\log\Big(\prod_{n=1}^{\infty}|\varphi_{X_n}(t)|^2\Big)\,dt = \frac{1}{h}\int_{|t|<h} -\log\big(|\varphi(t)|^2\big)\,dt < \infty.\end{aligned}$$

  This proves that the first sum in Theorem 5.5 converges.

  The second sum vanishes since we consider symmetric random variables.

  For the third sum we exploit (the second half of) Lemma 4.4.2 to obtain

$$\begin{aligned}\sum_{n=1}^{\infty} E|X_n^s|^2 I\{|X_n^s| < 2/h\} &\le \sum_{n=1}^{\infty} 3\big(1 - \varphi_{X_n^s}(h)\big) \le 3\sum_{n=1}^{\infty}\log\big(|\varphi_{X_n}(h)|^2\big) \\ &\le -3\log\Big(\prod_{n=1}^{\infty}|\varphi_{X_n}(h)|^2\Big) = -3\log\big(|\varphi(h)|^2\big) < \infty.\end{aligned}$$

- Page 505: Finally, let $a < 1 < b$. Since $\log a < 0$ in this case, the previous argument degenerates into

$$a\log^+ b \le a\log^+(b/a) \le a\log^+ a + b\cdot\frac{\log^+(b/a)}{b/a},$$

  and (9.1) follows again.