Probability: A Graduate Course

Misprints and Corrections

October 17, 2006

1. Misprints

Page	Line/Problem	Should be
vi	L25	chance exists
22	P4	Suppose also that $\{A_k\}$ are disjoint.
23	P5c	A_n, $n \geq 1$ are non-increasing
23	P5d	A_n, $n \geq 1$ are non-decreasing
23	P5d	then $P(\bigcup_{n=1}^{\infty} A_n) \leq \alpha$.
24	L5$_-$	*matching*
26	L7$_-$	$\mathbb{P}(\Omega)$
29	L2–7	These lines should be deleted.
29	L8	delete "Alternatively,"
30	L1$_-$	non-decreasing functions in D.
39	L5$_-$	trials
40	L17	$f(x) = \frac{\alpha\beta^{\alpha}}{x^{\alpha+1}}$
42	L7$_-$	$X \overset{d}{=} U$.
56	L6	Since $0 \leq 2X_1 - X_n \nearrow 2X_1 - X$
72	L2$_-$	the proof of The-
114	P5	$\sum_{n=-\infty}^{\infty} P(n < X \leq n+m) = m$
114	P6	and $a > 0$,
115	P8, L2	$\sup_n X_n < \infty \quad$ a.s.
116	P18	$\iff \quad \exists\, Z \in L^r$, such that $\lvert X_n\rvert \leq Z$ for all n
120	L11$_-$	$P(X > x) \leq$
123	L1$_-$	$B_k = \{\max_{1\leq j\leq k} \lvert S_j\rvert \leq x\}$
125	L13	$\sum_{k=1}^{n} \operatorname{Var} X_k \leq x^2 + \frac{\cdots}{\cdots}$. (. instead of , at the end)
127	L5$_-$	independent summands there exists
128	L7	$E\lvert X\rvert^r \leq c_r\big(E\lvert X+y\rvert^r + \lvert y\rvert^r\big) < \infty$.
129	L10	For $0 < p \leq r$
129	L11	$\lVert X\rVert_p \leq \lVert X\rVert_r$.
129	L13	(and q by $(1 - p/r)^{-1}$)
133	L18,19	The median is not unique in general, and typically
133	(ii)	for some $r > 0$,
133	(iii)	$\lvert \operatorname{med}(X) - E\,X\rvert \leq 2^{1/r}\lVert X - E\,X\rVert_r$.
139	L12$_-$	$= \frac{1}{2}P(\max_{1\leq k\leq n} S_k - \operatorname{med}(S_n - S_k) > x)$,
142	L10$_-$	$\lvert S_n - S_k\rvert \geq \ldots$

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Page	Line/Problem	Should be
150	$L7_{-}, 1_{-}$	Khintchine
155	P9b	$Q_X(ah) = aQ_X(h)$?
158	L2	$\varphi_X(t) = E\,e^{itX} = \int_{-\infty}^{\infty} e^{itx}\,dF_X(x)$
158	L13	$\|E\,e^{itX}\|$
159	L7	$\frac{pe^{it}}{1-qe^{it}}$
160	$L4_{-}, 2_{-}$	$H(a,b,x,T)$
160	$L3_{-}$	$H(a,b,X,T)$
161	L4	$H(a,b,x,T)$
161	L6	$H(a,b,X,T)$
164	L8	$E\exp\{it(X-a)\}$
165	$L8_{-}$	$\frac{1}{2} = \cdots = \frac{1}{\pi}\int_0^{\infty}\left(\frac{\sin x}{x}\right)^2 dx$
172	L14	(2.4)
184	L5	$-\frac{1}{n} \quad -\frac{1}{n} \quad 1-\frac{1}{n} \quad \ldots \quad -\frac{1}{n}$
195	$L8_{-}$	$f(x) = \frac{1}{\sigma x\sqrt{2\pi}}\exp\{\cdots\}$
195	$L6_{-}$	$\int_0^{\infty} \frac{C+2\log x+\frac{(2\log x-\mu)^2}{2\sigma^2}}{1+x^2}\,dx$
196	L2,4	delete 2π
196	L4,5	$\sum_{k=1}^{\infty}$
196	L5	$\geq C\sum_{k=1}^{\infty}\frac{1}{\beta+k}$
198	P12	$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1-\cos y}{y^2}e^{-\|y\|}\,dy = \frac{1}{2} - \frac{\log 2}{\pi}$
198	P14	$\int_0^{\infty}\varphi(tu)e^{-u}\,du$ or $\frac{1}{2}\int_{-\infty}^{\infty}\varphi(tu)e^{-\|u\|}\,du$
198	P14	$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\varphi(tu)}{1+u^2}\,du$
216	$L14_{-}$	let $Y_1, Y_2, \ldots$ be
216	$L8_{-}$	these facts
229	L 6	converges to a limit $a \in \mathbb{R}$ if and only if
229	L 7	contains a subsequence converging to a.
245	L 19	Theorem 10.2.
254	L9	$n \geq m$.
254	L13	$= \frac{1}{2}P\big(\bigcup_{n=m}^{\infty} A_n^{(m)}\big)$.
258	$L6_{-}$	$\inf\{x : \ldots\}$ (twice)
258	$L2_{-}$	$\ldots = F_n(x)$
261	P9, L2	converges in distribution
261	P10	$P(X_n < a$ i.o. and $X_n > b$ i.o.)
261	P11, L1	non-degenerate random
262	P13	$\sum_{n=1}^{\infty}\|a_n\| < \infty$
262	P16, L3	$\varphi(0) = 1$
263	P17	$\tilde{\varphi}(t) = \exp\big\{\int_{\mathbb{R}}\varphi(t,u)\,dG(u) - 1\big\}$
263	P21, L2	$n \geq 2$
263	P21, L3	Independence between $X_1, X_2, \ldots$ not necessary
264	P22(a)	$\iff \quad X_n^{(k)} \xrightarrow{p} X^{(k)}$
264	P22(b)	$\implies \quad h(\mathbf{X}_n) \xrightarrow{p} h(\mathbf{X})$
274	L17	For the converse: Assume also that $\frac{\mu_n}{b_n} \to 0$ as $n \to \infty$.

Page	Line/Problem	Should be
279	$L10_-$	$= \cdots \leq \frac{2}{n} + \frac{4}{n}\sum_{j=1}^{n} \cdots$
284	L12,14	$nE\,XI\{\lvert X\rvert \leq n\log_2 n\}$
286	$L2_-$	right-hand side
289	$L10_-$	symmetric and uniformly bounded
290	L11	bounded, symmetric
292	$L11_-$	Proposition 5.1.2
296	L2	$\limsup_{n\to\infty} \frac{\lvert S_n\rvert}{n}$
299	L3	$\sum_{n=1}^{\infty} \frac{Y_n - E\,Y_n}{n^{1/r}}$
299	L7	$\to 0$
299	L9	$\stackrel{a.s.}{\to} 0$
300	L1	$\leq Cn^{(1/2)-(1/2r)} \cdot E\lvert X\rvert^r$
300	L6	$\leq \sum_{k=1}^{n} E\lvert X_k\rvert I\{\lvert X_k\rvert > k^{1/r}\}$
303	$L2_-$	$Y_{N(t_{k_j})} \to Y$
305	$L11_-$	that, for any given $j = 0, 1, \ldots, k-1$,
305	$L10_-$	$\frac{1}{n}\sum_{i=1}^{n} I\{X_i = j\} \stackrel{a.s.}{\to} \frac{1}{k}$
309	L11	Theorem 5.4.1
310	L10	one can prove
324	L3	Elements a_{12} and a_{21} of $\mathbf{\Lambda}$ should be $\rho\sigma_x\sigma_y$ instead of ρ.
325	P9, L1	non-degenerate random
325	P11c	$\sum_{k=2}^{n}$
327	P22, L2	$Y_n = \max\{\ldots\}$
332	$L2_-$	$E\min\{\frac{2\lvert tX_k\rvert}{s_n}, \frac{t^2X_k^2}{2s_n^2}\}$
332	$L1_-$	$E\min\{\frac{t^2X_k^2}{s_n^2}, \frac{\lvert t\rvert^3\lvert X_k\rvert^3}{6s_n^3}\}$
335	L8	$X_n \stackrel{d}{\to} Y \iff E\,h(X_n) \to E\,h(Y)$
335	$L6_-$	X_j
338	L4,7	$\frac{X_k^2 t^2}{2s_n^2}$ (X_k^2 instead of σ_k^2)
343	L8	$P(Z_k = 0) = 1 - \frac{1}{k^2}$
344	$L2_-$	$S_n = \sum_{j=1}^{n} X_{n,j}$
345	L18	If $\sum_{j=1}^{n} E\,X_{n,j}^2 I\{\lvert X_{n,j}\rvert > \varepsilon\} \to 0$ as $n \to \infty$
345	L19	$S_n \stackrel{d}{\to} N(0,1)$
346	$L3_-$	$n_0 = [t]$
347	L14	$\frac{[n_0(1+\varepsilon^3)] - n_0}{\varepsilon^2 n_0}$
349	$L10_-, 7_-$	$n\big(\cdots\big) = \frac{1}{2}\big(\cdots\big)^2$
349	$L9_-$	Since x^2 is continuous
350	L9	$S_n = \sum_{k=1}^{n} X_k$
357	$L6_-$	$\mathrm{Tri}(-T,T)$-distributed
357	$L1_-$	$\ldots = \int_{-\infty}^{\infty} e^{ixt}\frac{2\sin^2(xT/2)}{\pi T x^2}\,dx = \ldots$
361	$L7_-$	r_k'
361	$L6_-$	$r_k' + r_k''$
361	$L5_-, 4_-$	θ_k'
362	L12	$\sup_x \lvert F_{S_n/s_n}(x) - \Phi(x)\rvert \leq \frac{16}{\pi}\frac{\beta_n^3}{s_n^3}\int_{-s_n^3/4\beta_n^3}^{s_n^3/4\beta_n^3} t^2 e^{-t^2/3}\,dt + \ldots$

Page	Line/Problem	Should be
376	P1,P2	$\iff \quad \sum_{k=1}^{\infty}$
377	P7, L2	$P\big(X_k = \frac{1}{\sqrt{k}}\big) = P\big(X_k = -\frac{1}{\sqrt{k}}\big) = 1/2$
377	P7, L4	$n \geq 2$
379	P17, L9	the amount of time (delete "relative")
379	P18, L2	$(S_n)^p$, suitably normalized,
380	P21	$\sqrt{n}\big(s_n^2 - \sigma^2\big)$
380	P23,L7	delete "suitably normalized".
385	L12	In other words, $\frac{S_n}{\sqrt{2\sigma^2 \log\log n}}$
396	L13	$n_k = \min\{n : s_n^2 > \lambda^k\}$ (which reduces to $[\lambda^k] + 1$ ...
399	$L8_-$	be a strictly
400	$L9_-$	be a strictly
436	L 15	$E\|X\|^r = \infty$ for $r > \alpha$. Both possibilities exist for $r = \alpha$.
436	L19	$a_n = \inf\{\ldots\}$
439	L9	$\int_1^{\infty} \frac{dy}{y}$
443	L1	infinitely divisible distribution
443	$L1_-$	due to (i)
450	L7,8,9,10	$\sup_{k \in \mathbb{Z}^+}$
451	$L3_-$	iff (one iff too many)
459	$L9_-$	$d(X,Y) \leq P(X \neq Y)$
459	$L8_-$	Let $X_1, X_2, \ldots$ and $Y_1, Y_2, \ldots$
464	P1	X is symmetric stable
465	P3,L1	X is symmetric stable
465	P3,L3	XY is symmetric stable
465	P5	$\sum_{n=1}^{\infty} \|a_n\|^{\alpha} < \infty$ (twice)
465	P7c	Incorrect. Find the correct representation. (Use (a))
470	L19	with respect to P; $Q \ll P$.
472	L3	$= \int_{\Lambda} \big(\cdots\big)\, dP.$
477	$L3_-$	$\{A_n,\, n \geq 0\}$
491	L2	arbitrary. (One . too much)
495	$L1_-$	$\int_{\Lambda} X_{\tau}\, dP = \int_{\Lambda} Z\, dP$ for all $\Lambda \in \mathcal{F}_{\tau}$.
498	L6	$X_n^{(1)}(\omega), \quad X_n^{(2)}(\omega),$ (, is missing)
500	L8	as long as $n < \tau$
503	L3	delete $\leq E\, X_n - E\, X_0$
503	L9	$\lambda P(\mathbf{\Lambda}) \leq \cdots = E\, X_n - \int_{\mathbf{\Lambda}} X_n\, dP - E\, X_0,$
504	L5	$\int_{\max\{0 \leq k \leq n\, c_k X_k^+ > \lambda\}} (\cdots) dP$
504	L5	$-c_n \int_{\{\max_{0 \leq k \leq n} c_k X_k^+ > \lambda\}} X_n^+\, dP$
505	L5	$= 1 + \int_{\Omega} X\big(\int_1^Y \frac{1}{y}\, dy\big)\, dP$
508	L20	almost surely
519	L4	$X_n^+ \overset{a.s.}{\to} X_{\infty}^+$
531	L24	triviality
532	$L1_-$	$\mu\tau$ (instead of $\mu(\tau)$)
536	L15	and (delete "that")

Page	Line/Problem	Should be
549	P7, L4	and that $E\|Y\|^r < \infty$
550	$L8_-, 7_-$	$n \geq 0$
551	P16	$X_n = E\big(A_\infty \mid \mathcal{F}_n\big) - A_n$
552	P25	$X_n = e^{S_n - n/2}$
552	P26	$W_n = \min\{Y_n, a\}$
560	$L1_-$	$-\sum_{n=1}^{\infty}$
564	$L6_-$	assume also that $B_n \nearrow \infty$ as $n \to \infty$
566	$L7_-$	non-zero exponent have an ultimately monotone equivalent.
572	$L7_-$	The line should be deleted.

2. Corrections

- The results in Chapter 8, except for Theorem 1.1, are for independent, *identically distributed* random variables (this is obvious(?) from the proofs).
- The reference to Kolmogorov on page 424 is not correct. It was after a lecture (one of three in a series) given in 1919 by Lévy himself that someone told him ... (The passage was somewhat hastily taken from the obituary in *Ann. Probab.* **1** (1973).)
- Problem 1, page 464. For this problem we also need to know that non-negative stable distributions with index in$(0, 1)$ possess a *Laplace transform.* In the present case this means that "we know" that

  $$L_Y(s) = E\,e^{-sY} = \exp\{-s^\beta\} \quad \text{for} \quad s > 0.$$

  For more about Laplace transforms, see Feller's Volume II, Chapter XIII, and for this kind of stable distributions, Section XIII.6.
- Problem 10, page 549: The second claim should be "the expected amount of money spent when the game is over". The third claim should be deleted. (The slip is due to a confusion with the St. Petersburg game, Subsection 6.4.1, page 283-4).