Probability: A Graduate Course, 2nd edition

Misprints and Corrections

21 oktober 2020

1. Misprints

Page	Line/Problem	Should be
26	$L10_-$	$P(R) = ...$ (not $P(\Omega)$)
33	$L5_-$	$\int_a^p g(x)d\gamma(x)$. (γ is missing)
57	$L12_-$	$\lvert\sum_{k=1}^n Y_k\rvert \le X$ for all n
75	L10	$r\sum_{n=1}^\infty \cdots \le E\,X^r \le 1 + r\sum_{n=1}^\infty \cdots$
76	$L5_-$	function, and that g' is monotone. Then
321	L9	[239] (not [238])
338	L1	However, the absolute value of the real part of the ...
341	$L2_-$	$L_1(n) = \cdots = \frac{1/2}{\cdots} \to \frac{1}{2} \ne 0$
342		The example on the lower piece has to be fixed
343	L15	$\sum_{n=1}^\infty P(\lvert Z_n\rvert > 0) = \cdots$
353	$L8_-$	$\overset{d}{\to}$ (not $\overset{r}{\to}$)
357	$L10_-$	$\sup_{x>0}(\lvert E\,X_k^3 I\{\lvert X_k\rvert \le x\}\rvert + xE\,X_k^2 I\{\lvert X_k\rvert\})$
361	L5, 11	Theorem 4.4.1 (not (4.4.1))
363	L4	$e^{1/18}$ (not $e^{5/144}$)
392	L9	$\sum_{k=k_0}^\infty P\Big(\Big\lvert \sum_{j=1}^k X_j I\{\lvert X_j\rvert > \frac{1}{2}b_{n_k}\}\Big\rvert > \eta\sqrt{n_k \log\log n_k}\Big) \le ....$
418	L4, 5	Williams' (not William's)
527	$L4_-$	$\alpha = P($Player B is the winner....) (not A)
527	$L3_-$	player A is the winner (not B)
528	L8	$\alpha = P($Player B is the winner....) (not A)

2. Corrections

Letter from J. Prochno, hereby acknowledged:

Lemma 3.1, Page 14 *Let $(\Omega, \mathcal{A}, \mu)$ be a finite measure space and $\mathcal{F}$ be an algebra on Ω with $\sigma(\mathcal{F}) = \mathcal{A}$. Then, for every $\varepsilon > 0$ and any $A \in \mathcal{A}$, there exists $B \in \mathcal{F}$ such that*

$$\mu(A \triangle B) < \varepsilon.$$

Proof. We define

$$\mathcal{D} := \big\{A \in \mathcal{A} : \forall\,\varepsilon > 0\,\exists\,A_\varepsilon \in \mathcal{F} : \mu(A \triangle A_\varepsilon) < \varepsilon\big\}.$$

Obviously, $\mathcal{F} \subseteq \mathcal{D}$, because for if $A \in \mathcal{F}$ and $\varepsilon > 0$, then we can just choose $A_\varepsilon := A \in \mathcal{F}$, which gives $\mu(A \triangle A_\varepsilon) = 0 < \varepsilon$. We now show that $\mathcal{D}$ is a Dynkin system. In view of the definition of a Dynkin system, we do this in three steps.

Step 1. $\Omega \in \mathcal{D}$, because $\mathcal{F}$ contains Ω, as it is an algebra and, as we saw before, $\mathcal{F} \subseteq \mathcal{D}$.

Step 2. Let $A \in \mathcal{D}$. We need to show that $A^c \in \mathcal{D}$. So let $\varepsilon > 0$. Since $A \in \mathcal{D}$, there exists some $A_\varepsilon \in \mathcal{F}$ so that $\mu(A \triangle A_\varepsilon) < \varepsilon$. Because $C \triangle D = C^c \triangle D^c$, we may choose $B_\varepsilon := A_\varepsilon^c \in \mathcal{F}$ (because $\mathcal{F}$ is an algebra) and obtain

$$\mu(A^c \triangle B_\varepsilon) = \mu(A^c \triangle A_\varepsilon^c) = \mu(A \triangle A_\varepsilon) < \varepsilon.$$

This means $A^c \in \mathcal{D}$.

Step 3. Let $(A_i)_{i\in\mathbb{N}} \in \mathcal{D}^{\mathbb{N}}$ be a sequence of (pairwise disjoint) sets. We need to show that $A := \bigcup_{i=1}^{\infty} A_i \in \mathcal{D}$, that is, if $\varepsilon > 0$, then we need to find a set $B_\varepsilon \in \mathcal{F}$ so that $\mu(A \triangle B_\varepsilon) < \varepsilon$. So let $\varepsilon > 0$. First of all, by the continuity from below, there exists an $n_\varepsilon \in \mathbb{N}$ so that

$$\mu\Big(A \setminus \bigcup_{i=1}^{n_\varepsilon} A_i\Big) < \frac{\varepsilon}{2}\,. \tag{\textbf{S}.0.1}$$

In fact, the sequence $\bigcup_{i=1}^{n} A_i$, $n \in \mathbb{N}$ is increasing and $\bigcup_{i=1}^{n} A_i \uparrow A$, that is, $\lim_n \bigcup_{i=1}^{n} A_i = A$, which means that by the continuity from below, there exists $n_\varepsilon \in \mathbb{N}$ so that

$$\mu(A) - \mu\Big(\bigcup_{i=1}^{n_\varepsilon} A_i\Big) < \frac{\varepsilon}{2}\,.$$

Therefore, from the basic properties of a measure, it follows that

$$\mu\Big(A \setminus \bigcup_{i=1}^{n_\varepsilon} A_i\Big) = \mu(A) - \mu\Big(\bigcup_{i=1}^{n_\varepsilon} A_i\Big) < \frac{\varepsilon}{2}\,.$$

Now, since $(A_i)_{i\in\mathbb{N}} \in \mathcal{D}^{\mathbb{N}}$, for $\delta := \frac{\varepsilon}{2n_\varepsilon}$ there exists a sequence $(A_{i,\delta})_{i\in\mathbb{N}} \in \mathcal{F}^{\mathbb{N}}$ so that, for all $i \in \mathbb{N}$,

$$\mu(A_i \triangle A_{i,\delta}) < \delta. \tag{\textbf{S}.0.2}$$

Because

$$\Big(\bigcup_{i=1}^{n_\varepsilon} A_i\Big) \triangle \Big(\bigcup_{i=1}^{n_\epsilon} A_{i,\delta}\Big) \subseteq \bigcup_{i=1}^{n_\varepsilon} (A_i \triangle A_{i,\delta}),$$

we obtain from Inequality **S**.0.2,

$$\mu\Big(\Big(\bigcup_{i=1}^{n_\varepsilon} A_i\Big) \triangle \Big(\bigcup_{i=1}^{n_\varepsilon} A_{i,\delta}\Big)\Big) \le \sum_{i=1}^{n_\varepsilon} \mu\big(A_i \triangle A_{i,\delta}\big) < n_\varepsilon \delta. \tag{\textbf{S}.0.3}$$

It now follows from Inequalities (**S**.0.1) and (**S**.0.3) that, choosing $B_\epsilon := \bigcup_{i=1}^{n_\epsilon} A_{i,\delta}$,

$$\mu\Big(A \triangle B_\varepsilon\Big) < \frac{\varepsilon}{2} + n_\varepsilon \delta = \frac{\varepsilon}{2} + n_\varepsilon \frac{\varepsilon}{2n_\varepsilon} = \varepsilon.$$

This shows that $A \in \mathcal{D}$ and completes the proof of the theorem. □