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Analytic Number Theory, 1MA038

Omtentamen, 2016-03-22

Inga hjälpmedel. (Just paper and pen. No calculators, books, etc.)

1. Define: a) the graph Laplacian, b) Dirichlet convolution. And c) give the statement of the prime number theorem.
2. Formulate and prove Euler's product formula for Riemann's zeta function.
3. Calculate the logarithmic derivative of the gamma function in $s = 1/2$.
4. Explain and prove how certain 2×2 matrices define complex automorphisms (also known as bi-homomorphisms) of the upper half plane.
5. Determine all Dirichlet characters mod 5.
6. We know for suitable τ och z the formula:

$$\prod_{n=1}^{\infty} (1 - e^{2n\pi i\tau})(1 + e^{(2n-1)\pi i\tau} e^{2\pi iz})(1 + e^{(2n-1)\pi i\tau} e^{-2\pi iz}) = \sum_{n=-\infty}^{\infty} e^{\pi i n^2 \tau} e^{2\pi i n z}.$$

Based on this, prove that for appropriate x one has

$$\prod_{n=1}^{\infty} (1 - x^n) = \sum_{k=-\infty}^{\infty} (-1)^k x^{k(3k+1)/2}.$$

Explain one application of this formula to number theory.