Uppsala University
Department of Mathematics
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Examination in mathematics
2022-01-11

Analytic Number Theory

Time: 8.00 – 13.00. No tools are allowed except paper and pen.

1. Define the class number $h(d)$ which we studied in the course. (5p)

2. Outline a proof of the fact that if χ is a non-principal Dirichlet character then the Dirichlet series $\sum_{n=1}^{\infty} \frac{\chi(n)}{n^s}$ converges for all $s \in \mathbb{C}$ with $\Re(s) > 0$. (5p)

3. Define the Γ-function, and explain how this function provides a meromorphic continuation to all $\mathbb{C}$ of the arithmetic function $n \mapsto n!$. (5p)

4. State Rényi's large sieve bound (that is, a certain bound on the cardinality of a set of integers, when we assume that this set is contained in a certain interval, and satisfies certain congruence conditions). (5p)

5. Outline a proof of the prime number theorem.
(Note: You do not need to prove anything, instead merely describe the main steps of a proof.) (10p)

6. State the triple product formula for $\Theta(z \mid \tau)$, and outline a proof of this formula. (That is, describe the main steps of a proof.) (10p)

LYCKA TILL / GOOD LUCK!