Elementär Talteori: 2015-06-10

Hjälpmedel: Papper och skrivdon, miniräknare.

1. Determine all integer solutions of the equation:

   (a) $6x - 9y = 3$,

   (b) $2x - 2y + 4z = 2$.

2. Solve the following equations in the residue class ring $\mathbb{Z}_n$!

   (a) $163x = 1$, where $n = 221$

   (b) $x^2 = x$, where $n = 105$

   (c) $x^7 - 14x - 2 = 0$, where $n = 49$.

3. Let $n = ab$. The Chinese remainder theorem states that the homomorphism $\psi : \mathbb{Z}_n \longrightarrow \mathbb{Z}_a \times \mathbb{Z}_b$ med $\psi(x + \mathbb{Z}n) := (x + \mathbb{Z}a, x + \mathbb{Z}b)$ is an isomorphism, if $\gcd(a, b) = 1$. Show that that condition is necessary as well.

4. Find all solutions $(x, y) \in \mathbb{N}^2$ of the equation $x^2 + y^2 = 1768$.

5. (a) Show that $\overline{2} \in \mathbb{Z}_{29}^*$ is a primitive root.

   (b) Solve the equation $x^2 = -1$ in $\mathbb{Z}_{29}^*$!

6. Has the quadratic equation $x^2 + 545x + b = 0$ a solution in $\mathbb{Z}_{557}$ for $b = 210, 673$? (557 is prime.)

7. Write $\frac{2014}{1945}$ and $\sqrt{8}$ as a continued fraction!

8. Show that the equation $x^2 - 10y^2 = 54$ has infinitely many solutions $(x, y) \in \mathbb{N}^2$. Determine two of them!

Lycka till!