Condorcet's jury principle

Many of our decisions in life are made after talking to other people. We ask our friends, family and work colleagues for advice, often consulting widely to get the best information possible. Consultation is not only important in our personal life, but it is the bedrock of modern democracy. The last 100 years has proved time and again that democracy works better than dictatorship. Consensus is better than by leadership of a single person. Why is it that decisions made by large groups are better than those made by one person alone?

Understanding of the world around us can usually be sharpened by mathematics, and it was to mathematics that the French intellectual Marquis de Condorcet turned to at the end of the 18th century. Condorcet's political ideas were guided by the looming revolution in his country. He was an advocator of a liberal economy, equal rights and free public education. He was also a mathematician, inheriting his knowledge from the foundational work on probability by Pascal, Fermat and Bernoulli. In 1785 he published a work which combined his political aims of the future with a use of mathematics from hundred years earlier. His '*Essay on the Application of Analysis to the Probability of Majority Decisions'* gave, amongst many ideas tying together mathematics and political science, a theory of why juries consisting of many individuals are likely to reach better decisions than single 'experts'.

Condorcet's argued as follows. Assume that an odd number of individuals n have to make a choice between two options. These choices are made independently of the others and each individual has a probability p of being correct. The probability that the majority make the correct choice follows directly from the derivation of the binomial distribution. In particular, the probability a majority are correct is

$$m(n,p) = \sum_{i=\frac{n+1}{2}}^{n} \binom{n}{i} p^i (1-p)^i$$

Figure 1 plots this function for p=0.6. As the number of individuals goes to infinity, $m(n,p) \rightarrow 1$ and the majority decision is always correct. In the case where n is an even number we have to make a choice about how we treat cases where an equal number make the same decision, but the overall shape of the curve remains the same.

Let us compare our 'jury' to a single expert in a modern day context. Imagine a nation with a population of 300 million who have to elect a new president. Let us assume these people aren't so bright and when asked any question the chance of getting a correct answer out of them is 50.01%, not much more than fifty-fifty. Should we trust these people to elect their president or should we send Carl Bildt there to do it for them? Substituting these values for n and p in to $m(n,p)$ we find that Mr Bildt would have to be 99.97% sure of making the correct choice for him to override the people of this country. Evidence indeed of the power of democracy.

Or is it? Although Condorcet's jury principle seems to provide a powerful way for groups to make correct decisions it relies on two key assumptions: individuals are unbiased and independent (they behave like random variables!). Both these assumptions must be treated with care. If we deliberately mislead people then no matter how many subjects we consult they will all make the wrong choice. But worse still, there doesn't have to be any deliberate campaign of misinformation in order for wrong choices to be made. The crucial assumption

of independence depends on people not consulting each other before they vote. Hardly, a reasonable assumption for a modern day election.

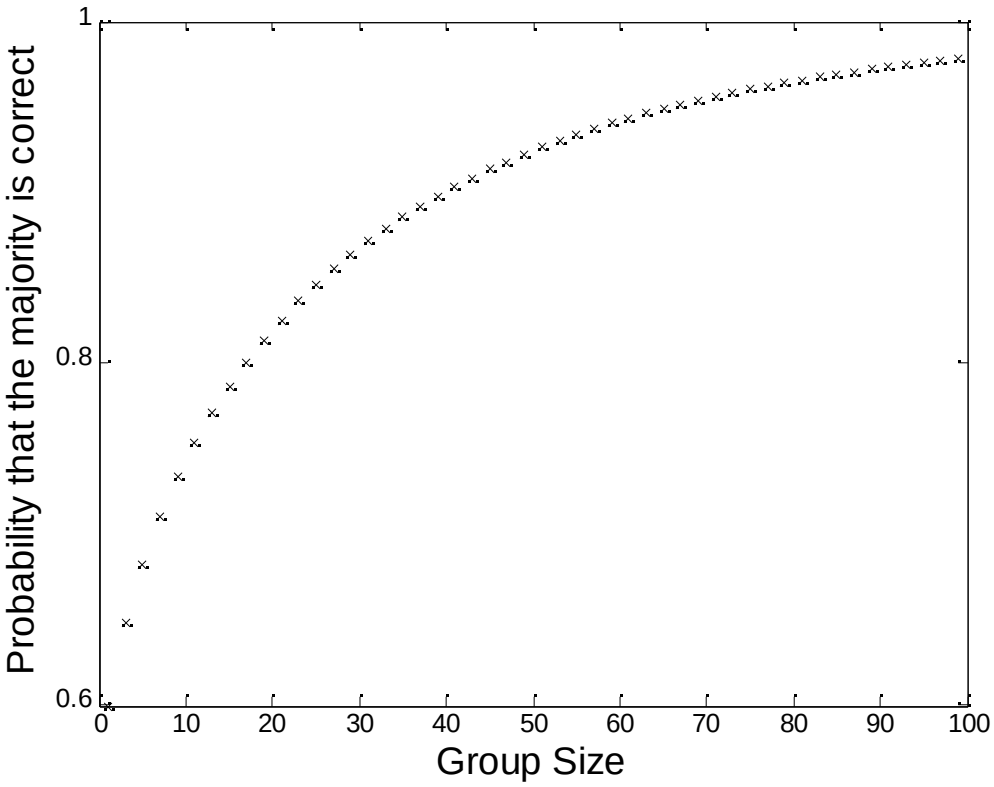

Figure 1: The probability that the majority of individuals are correct (for odd numbers of individuals) when each is correct with probability p=0.6.

Condorcet learnt the importance of these assumptions when only four years after the publication of his essay the French revolution gave him the chance to put his political ideas in to practice. The French king was removed from power and Condorcet was elected as a member of the new republic's assembly in 1791. But soon after his election the assembly broke in to two factions, with opposed views of how the revolution should proceed. Condorcet was declared a traitor, imprisoned and two days later was found dead in his cell. A bitter irony for the man who offered a mathematical proof of democracy.

It would be a shame to end on this note. So I'll say this instead. Mathematics is a method from taking us from a set of assumptions to a set of consequences. While our world doesn't always obey our assumptions and therefore it doesn't always follow our consequences, mathematics is the only way we know for making such arguments rigorously. For this reason Condorcet's arguments are true and useful today and will always remain so.

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