Expected recoveries in period dt

$$p\,x(t)$$

Expected number of infected contacting someone else is

$$q\,x(t)$$

The probability the contacted person is not infected is approximately

$$\frac{N - x(t)}{N\,\cancel{x(t)}}$$

From this

$$x(t+dt) \approx \cancel{p}\,x(t) - p\,\cancel{t}\,x(t) + q\,x(t)\cdot\frac{N - x(t)}{N\,\cancel{x(t)}}$$

$$\frac{x(t+dt) - x(t)}{dt} \approx -p\,x(t) + q\,x(t)\cdot\frac{N - x(t)}{\cancel{x(t)}\,N}$$

$dt \to 0$

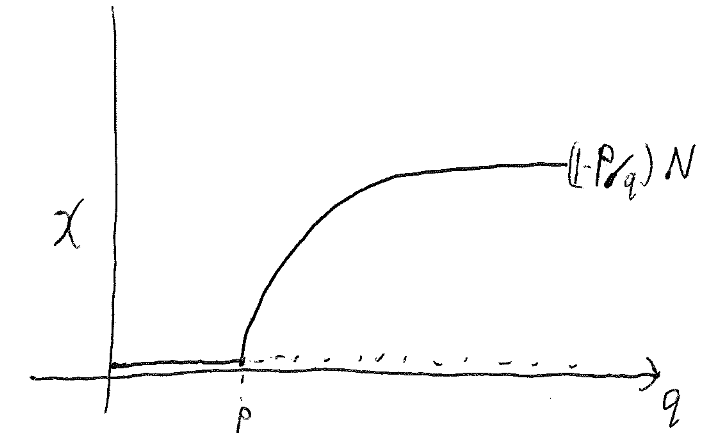
$$\frac{dx}{dt} = -px + qx\left(1 - x/N\right) \equiv f(x)$$

At eqm

$$\frac{dx}{dt} = 0 \qquad px = qx\left(1 - x/N\right)$$

$$\Rightarrow \quad x = 0 \quad \text{or}$$

$$p/q = 1 - x/N \qquad x = \left(1 - p/q\right)N$$



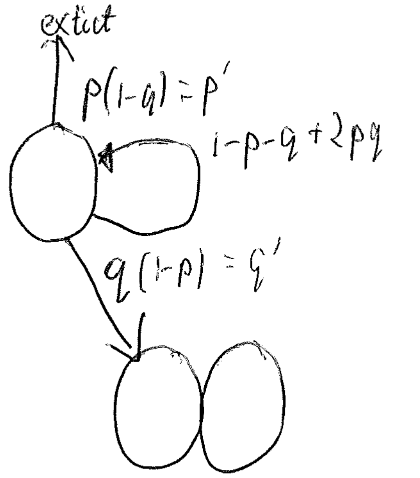
$$f'(x) = -p + q\left(1 - 2x/N\right)$$

$$f'(0) = q - p \Rightarrow \text{stable if } p > q.$$

$$f'\left(\left(1 - P/q\right)N\right) = -p + q\left(\cancel{1} - \cancel{2} + 2P/q\right)$$

$$= p - q \Rightarrow \text{stable if } q > p.$$

Let s be the probability of ultimate extinction.
~~If we have~~ starting with $x(t) = 1$ infected individual.



$$s = p' \cdot 1 + (1 - p' - q') \cdot s + q'\frac{(N-1)}{N} s^2$$

both ind must go extinct

$$\approx p' + (1 - p' - q')s + q's^2$$

$$0 \approx p' - (p' + q')s + q's^2$$

Solve for s

$\cancel{s = \frac{(p+q) \pm \sqrt{(p+q)^2 - 4pq}}{2q}}$

$s = 1$

or $s = p'/q'$

probability generating functions, Markov chains