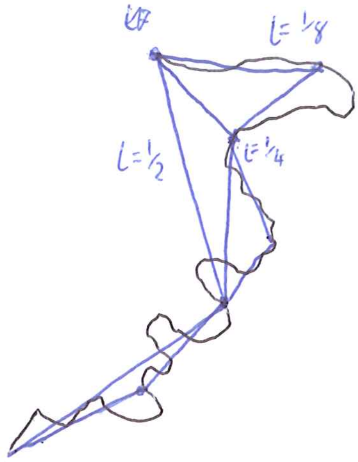
Fractal Dimension



How long is the coast of the U.K. depends on how long your ruler is!

Cantor Set

$i=0$ $N_i=1$ $l=1$

$N_i=2$ $l=\frac{1}{3}$

$N_i=4$ $l=\frac{1}{9}$

$N_i=8$ $l=(\frac{1}{3})^3$

$N_i=16$ $l=(\frac{1}{3})^4$

remove middle third

$$N_i = 2^i \quad , \quad l = \left(\frac{1}{3}\right)^i$$

Box counting dimension is d iff positive k s.t.

$$\lim_{l\to 0} \frac{N_i}{1/l^d} = k$$

$$\lim_{l\to 0} \left(\ln(N_i) + d \ln(l) \right) = \ln(k)$$

$$\lim_{l\to 0} \left(\frac{\ln(N_i)}{\ln(l)} \right) + d = \underbrace{\cancel{\lim_{l\to 0} \frac{\ln(k)}{\ln(l)}}}_{0}$$

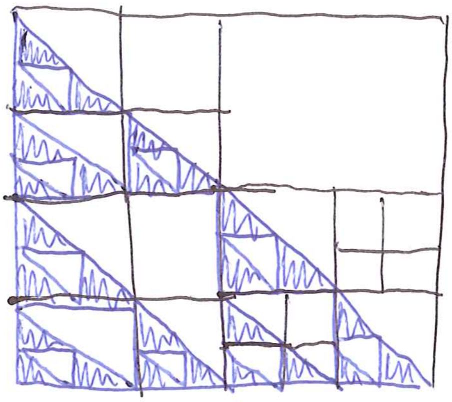
$$d = -\lim_{l\to 0} \left(\frac{\ln(N_i)}{\ln(l)} \right)$$

For Cantor set:

$$N_i = 2^i \qquad l = \left(\tfrac{1}{3}\right)^i$$

$$d = \frac{\ln(2)}{\ln(3)}$$

Serpinski Gasket



$N_i = 1 \qquad l = 1$

$N_i = 3 \qquad l = 1/2$

$N_i = 9 \qquad l = 1/4$

$N_i = 27 \qquad l = 1/8$

$N_i = 3^i \qquad l = 2^{-i}$

N_i - number of boxes with 'something' in it.

l = length of side of box

$$d = \frac{\ln(3)}{\ln(2)}$$

Power laws

s - size of event

$P(s)$ - prob. of event of ~~the~~ size s.

Assume s is <u>scale invariant</u>. We make an abitrary scale change λs and

$$\mu P(s) = P(\lambda s).$$

i.e. Events of size ~~S~~ are μ times more likely than events of size λs for all λ.

Solution is:

$$P(s) = C s^d$$

Check: $\nu \cancel{C} \cancel{s^d} = \cancel{C} \lambda^d \cancel{s^d}$

$$d = \frac{\ln(\nu)}{\ln(\lambda)}$$

$$\ln(P(s)) = \ln(c) - d\ln(s)$$