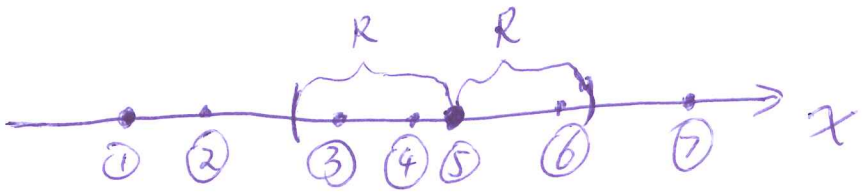


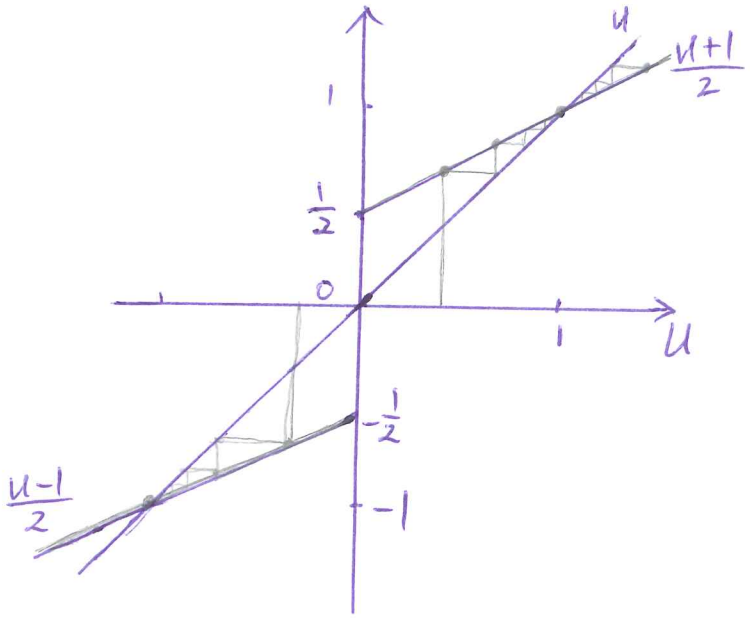
$$f(x) = \mathrm{sign}(x) = \begin{cases} 1 & \text{if } x>0 \\ -1 & \text{if } x<0 \\ 0 & \text{if } x=0 \end{cases}$$

R: interaction range

Take $i=5$ as an example

$N_5(t) = 3 \quad (3, 4, 6)$

$$S_5(t) = \frac{1}{N_5}\sum_{j=1}^{3} \mathrm{sign}(x_j - x_5) = \frac{1}{3}[-1 - 1 + 1) = -\frac{1}{3}$$



$$G(u) = \begin{cases} \frac{u+1}{2} & \text{if } u>0 \\ \frac{u-1}{2} & \text{if } u<0 \end{cases}$$

Φ equilibrates at either -1 or 1

For every particle $\vec{v}_i = v_0 \cdot \vec{u}_i$

v_0: absolute value of the velocity

$\vec{u}_i$: a unit vector shows the direction of the velocity

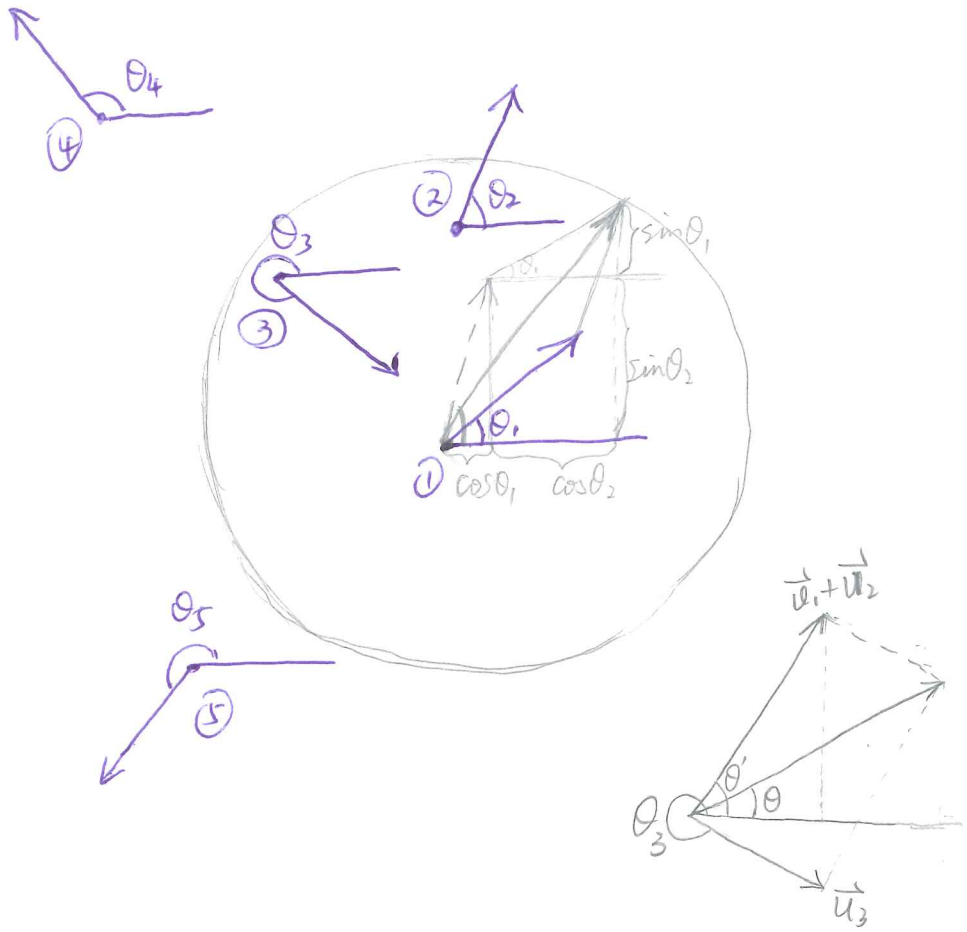
If all the particles go in the same direction

$$\sum_{i=1}^{N} \vec{v}_i = Nv_0 \Rightarrow \Phi = 1 \text{ or } -1$$

or $-Nv_0$

If all particles move in random directions

$$\sum_{i=1}^{N} \vec{v}_i \approx 0 \quad \Rightarrow \quad \varphi \approx 0$$



If we ignore the random noise, then

$$\theta(t+1) = \langle \theta(t) \rangle_i$$

$$\Updownarrow$$

$$\vec{v}_i(t+1) = \frac{1}{N_i} \sum_j \vec{v}_j$$

$$= \frac{v_0}{N_i} \sum_j \vec{u}_j$$

$$\tan\theta = \frac{(\sin\theta_1 + \sin\theta_2) + \sin\theta_3}{(\cos\theta_1 + \cos\theta_2) + \cos\theta_3}$$

$$= \frac{\sum_{j=1}^{3} \sin\theta_j}{\sum_{j=1}^{3} \cos\theta_j}$$

$$\text{so} \quad \theta = \arctan\left(\frac{\sum_{j=1}^{3} \sin\theta_j}{\sum_{j=1}^{3} \cos\theta_j} \right)$$