First computer lab.

- 1. Write a program that computes the machine epsilon.
- 2. Implement Kahan summation algorithm and discuss its accuracy with examples.
- 3. Write a program that switches the rounding mode of your computer.
- 4. Consider the infinite series

$$\zeta(s) = \sum_{k \ge 1} \frac{1}{k^s}.$$

Compute rigorous lower and upper bounds of $\zeta(s)$ for s = 2, 3, 4, 5, 6. Are your results tight?

5. Compute rigorous lower and upper bounds of the integrals

$$\int_{0}^{1} e^{\sin(x^{2})} dx; \\ \int_{1}^{\infty} \sin(x^{2}) x^{-3} dx.$$

6. How many times does the function

$$\sin(x)\cos(x) + \frac{1}{2} - (x+1)\sin\left(\left(x+\frac{1}{2}\right)^2\right) + \exp(-x)$$

vanish in the interval [0, 2]?