UPPSALA UNIVERSITET
Matematiska Institutionen
Validated Numerics. Fall 2015.
Jordi-Lluís Figueras (figueras@math.uu.se)

First set of Problems. First part.

1. Compute $(1.01)_2^{(1.21)_3}$.

2. In 1958 Moscow State University developed a *balanced ternary* computer called **Setun**. Balanced ternary is a base 3 numeral system using the digits $-1, 0, 1$.

   (a) Explain how to add, substract, multiply and divide two numbers in this representation.

   (b) Suppose you have a two-pan balance and only one weight for all powers of 3. Interpret how to weight a body in terms of the balanced ternary system.

   (c) Convert the following numbers to balanced ternary: $10, -4, 1/2, -13.4$.

   (d) Think of how you would represent balanced ternary computer representable floating point numbers. Do you see any advantage in this system? What about representing numbers in the standard base 3?

3. Notice that $19 = (19)_{10} = (10011)_2 = (201)_3$. What base is more economical for representing numbers? To answer this question, consider the *economy* function

$$E(b, N) = b\lfloor \log_b(N) + 1\rfloor.$$

   This function is supposed to measure the cost of storing number N in base b. Explain this last assertion and find the best base.

4. Let $\{x_i\}_{0\leq i<N}$ be a sequence of computer representable floating point numbers. There are two ways of computing their sum $S = \sum x_i$:

   (a)
```
s = 0;
for(i = 0; i < N; i++)
{
   s = s+x[i];
}
//s is the solution
```

   (b) **(Kahan summation algorithm)**.
```
s = 0;
c = 0;
for(i = 0; i < N; i++)
{
   y = x[i]-c;
   t = s+y;
   c = (t-s);
   c = c-y;
   s = t;
}
//s is the solution
```

   Which one is better? Discuss both.

5. One way of increasing the precision of how to represent numbers is by combining several `doubles`. Let x, y be two non-overlapping `doubles` (x and y do not overlap if there exists integers r and s such that $x = r2^s$ and $|y| < 2^s$ or $y = r2^s$ and $|x| < 2^s$). Then, $x + y$ represents a number with larger precision. How will you add two `double-doubles`? More precisely, given the two `doubles` a, b, how will you find non-overlapping `doubles` x, y such that $a + b = x + y$?

6. We have discussed during the lectures how to represent $\mathbb{R}$ with a computer. Explain how you will represent the following spaces: $\mathbb{R}^n$, $\mathbb{C}^n$, $\mathcal{C}([a, b], \mathbb{R})$.