Homework set.
Crash Course C.
2014–11–18

Instructions: You must solve the exercises individually. Submit all codes.
Deadline: 18 January 2015.

1. Compute all prime numbers below N. Check the prime number theorem.

2. Compute all integral solutions of the inequality

$$x^2 + y^2 \leq N.$$

3. Consider a domain $\Omega \in \mathbb{R}^2$ with two boundaries $\delta\Omega_i$, $i = 0, 1$. Compute $u(p)$, $p \in \Omega$, solving the problem

$$\begin{cases} \Delta u = 0 \\ u_{|\delta\Omega_i} = i \end{cases}$$

using a Monte-Carlo method.

4. Consider the trigonometric polynomial $f : \mathbb{T}^2 \to \mathbb{R}$,

$$f(t) = \sum_{|k| \leq N} c_k e^{2\pi i k t}.$$

Find all extrema and classify them (Morse index).

(Extra) Compute the attached invariant manifolds of the points.

5. Integrate the PDE

$$u_t = \Delta u + F(u)$$

defined on the unit interval with Dirichlet conditions

$$u(0, t) = p_1(t), u(1, t) = p_2(t),$$

where p_i are 1 periodic functions.

Choose between two types of discretized versions: Fourier expansions or finite differences.

6. Consider the functional equation

$$f(g(z)) = g(\lambda z),$$

where $\lambda = 0.5$, $f(z) = \lambda z + z^2$ and g is an (unknown) analytic function with $g(0) = 0$. Compute its n-th Taylor expansion.