UPPSALA UNIVERSITET
Matematiska Institutionen
Statistics for biology. Spring semester 2015.
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First set of exercises:
Graphs. Sample means, variances and standard deviations. Probability. Discrete random variables. The binomial distribution.

1. Given the following sample, compute its range, mean, median, mode, variance and standard deviation.

   1 2 2 4 5 3 7 8 -2

   Answer: Range=[-2,8], Mean=3.333, Median=3, Mode=2, Variance=9.5, Standard Deviation=3.082207.

2. The following list corresponds to a sample of the annual salary of randomly chosen citizens.

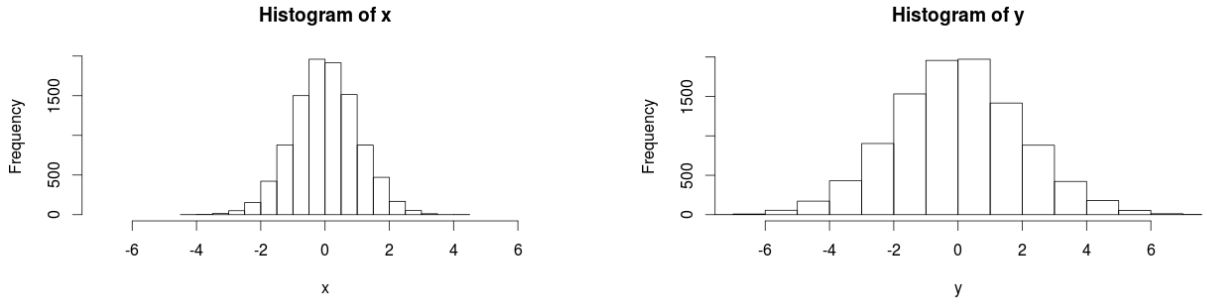
210 000 SEK	150 000 SEK	200 000 SEK	180 000 SEK
230 000 SEK	1 000 000 SEK	10 000 000 SEK	180 000 SEK

   Compute its mean and median. Is it fair to summarize the sample by just using the mean? Compute its quantiles.

   Answer: Mean=1 518 750, Median=205 000. Quantiles: 1st=180 000, 2nd=205 000, 3rd=615 000.

3. [⋆] Today, the ages of a family of five members has sample mean 24 and standard deviation 14.5. Compute the sample mean and deviation after a decade, and after two decades. Explain your results. Explain how their histograms change with these translations.

4. Consider the following two histograms:



   Both histograms represent data with the same mean. Could you estimate it? Could you also estimate the median, mode, quantiles? Can you compare their standard deviations?

5. Draw the histograms and cumulative graphs of the data given in the first two exercises.

6. Let X be the sum of the results of two dice. Describe the sample space Ω and the probability for every $x \in \Omega$.

7. Let X be the result of a die and Y the result of a second die. Describe the sample space Ω and the probability for every $x \in \Omega$.

8. Let X and Y represent the random variables described in the previous exercise. Are they independent? If we know that $X + Y$ is always equal to 10, are they independent?

9. Let X be the result of a die, Y the result of a second die and $Z = X + Y$. Compute: $P(Z < 2), P(2 < Z < 4), P(2 \leq Z \leq 4)$.

10. Consider the following table

Björn	Agnetha	Benny	Frida
10	12	20	22

    This table represents the amount of SEK that Björn, Agnetha, Benny and Frida have in their pockets. Let X be a discrete random variable that represents checking the amount of one of their pockets at random.

    - Compute: $P(X < 30), P(X < 14), P(10 \leq X)$.
    - Specify the (cumulative) distribution function $F(x) = P(X \leq x)$.
    - Sketch F.

11. A coin is biased with probability of getting heads equal $p = 0.2$. What is the probability of getting 4 heads if I casted it 4/5/20 times?

12. Let X be a discrete random variable that satisfies $P(X = 0) = 0.2, P(X = 3) = 0.5$ and $P(X = -4) = 0.3$. Compute its expected value, variance and standard deviation.

13. Let X be a discrete random variable that satisfies $P(X = -1) = 0.1, P(X = 0) = 0.1$ and $P(X = 1) = 0.8$. Compute $E(X), E(X^2), E(|X|), \sigma^2(X), \sigma(X)$.

14. The probability of being left handed is $p = 0.1$. What is the probability that a group of 20 has exactly two left handed?

15. The probability of being left handed is $p = 0.1$. What is the probability that a group of 20 has a left handed?

16. [$\star$] Yesterday I was doing the laundry and washed all my socks (I have a lot of socks!). Because not all of them fit in the washing machine, I just put 20. Afterwards, I realized that out of the 20 socks, 16 where left-footed. May I conclude that I have lost some of them? ($\alpha = 0.05$).

17. We test an attribute in an experiment with 20 individuals and found that 18 are positive (so just 2 are negative). Perform a hypothesis test with probability $p = 0.4$. ($\alpha = 0.05$).