UPPSALA UNIVERSITET
Matematiska Institutionen
Statistics for biology. Spring semester 2015.
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Second set of exercises: Continuous random variables. The normal distribution.

1. Consider the following function

$$f(x) = \begin{cases} x & \text{if } x \in [-2, 2] \\ 1 & \text{if } x \in (2, 3) \\ 0 & \text{otherwise} \end{cases}$$

   Is it a density function of a continuous random variable?

2. Consider the following function

$$f(x) = \begin{cases} 2 & \text{if } x \in [-2, 2] \\ 0 & \text{otherwise} \end{cases}$$

   Is it a density function of a continuous random variable?

3. Let X be a random variable with density function

$$f(x) = \begin{cases} 1 & \text{if } x \in [2, 3] \\ 0 & \text{otherwise} \end{cases}$$

   Compute its expected value, variance and standard deviation.

4. Let X be a random variable with density function

$$f(x) = \begin{cases} 3x^2 & \text{if } x \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

   Compute $E[X]$, $E[2X + 3]$, $Var(X)$ and $Var(4X + 2)$.

5. Let X be a random variable with density function

$$f(x) = \begin{cases} e^{-x} & \text{if } x \in (0, +\infty) \\ 0 & \text{otherwise} \end{cases}$$

   Compute $E[X]$ and $Var(X)$. Sketch its distribution function.

6. Let X be a random variable with density function

$$f(x) = \begin{cases} \frac{1}{2}xe^{-x} & \text{if } x \in (0, +\infty) \\ 0 & \text{otherwise} \end{cases}$$

   Compute $E[X]$ and $Var(X)$. Sketch its distribution function.

7. Let X be a random variable with density function

$$f(x) = \begin{cases} \frac{1}{2} & x \text{ if } \in [2,4] \\ 0 & \text{otherwise} \end{cases}$$

Compute $P(X < 3)$ and $P(2.5 < X < 3.5)$.

8. Let X be a random variable with density function

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 2x & \text{if } x \in (0, \frac{1}{2}) \\ 1 & \text{if } x \in [\frac{1}{2}, \frac{5}{4}) \\ 0 & \text{if } x \geq \frac{5}{4} \end{cases}$$

Compute its expected value, variance and standard deviation. Compute $P(X < 0.3)$ and $P(X > 0.5)$.

9. Let X be a normally distibuted random variable with $X \sim \mathcal{N}(4,2)$. Is $Y = 3X + 2$ normally distributed? Compute $E[Y]$ and $Var(Y)$.

10. Let X_1, X_2 be independent normally distibuted random variables, with $X_1 \sim \mathcal{N}(0,2)$ and $X_2 \sim \mathcal{N}(1,3)$. Is $X_1 + X_2$ normally distributed? Compute its expected value and variance.

11. We know that if $X \sim \mathcal{N}(0,1)$, then $P(X < 1.644854) = 0.95$ (approximately). Let $Y = 3X+2$. For which value α, $P(Y < \alpha) = 0.95$?

12. The IQs of students of a high school are normally distributed with expected value equal 125 and standard deviation equal to 10. What is the minimum IQ of the top 0.05? And the maximum IQ of the lowest 0.05?