UPPSALA UNIVERSITET
Matematiska Institutionen
Statistics for biology. Spring semester 2015.
Jordi-Lluís Figueras (`figueras@math.uu.se`)

Third set of exercises: Confidence intervals and hypothesis tests.

1. Let X_1 and X_2 be two random variables with the same mean μ. Instead of computing the sample mean using the formula
$$\frac{1}{2}(X_1 + X_2),$$
we want to use one of these two:
$$0.2X_1 + 0.8X_2,$$
$$0.3X_1 + 0.5X_2.$$
Are they biased?

2. Let X_1 and X_2 be two random variables with mean μ and variance σ^2. Compute $E[\frac{1}{2}((X_1 - \bar{X})^2 + (X_2 - \bar{X})^2)]$, where $\bar{X} = \frac{1}{2}(X_1 + X_2)$. Is it equal the σ^2?

3. We perform 20 times an experiment. We know that the population is normally distributed with $\sigma = 10$, but with unknown μ. If from these 20 experiments we get a sample mean equal 1.1, could you give a confidence interval for the population mean? The confidence interval should be computed with confidence coefficient 0.95.

4. With the same data as in the previous exercise, test if we can accept that the mean is equal $\mu_0 = 1$.

5. We take blood samples from 10 patients and measure the urea concentration on them. We get that the sample mean is 6.3 $mmol/L$ and the sample standard deviation is 1.1. It is known that this concentration behaves as a normally distributed random variable and, for a healthy person, the mean should be 4.45 with standard deviation 0.975. Could we say that the patients have a normal urea concentration in their blood? Or is it higher?

6. An oil company wants to determine the mean weight of a can of its oil. It takes a random sample of 80 such cans (from several thousand cans in its warehouse), and finds the (sample) mean weight is 31.15 ounces and the (sample) standard deviation is 0.08 ounces.
   - Compute a 95 percent interval for the mean weight of the cans in the firm's warehouse.
   - Is your previous answer based on the assumption that the weights of the cans of oil are normally distributed? Why, or why not?

7. A school board is responsible for two elementary schools. It wants to determine how the mean IQ of the students at school *Gryffindor* compares with the mean IQ of those at school *Slytherin*. It chooses a random sample of 90 students from each school. At Gryffindor the sample mean IQ is 109, and at Slytherin it is 98.

   - Compute a 95 percent confidence interval if we know that both schools have the same standard deviation, which is equal to 5.
   - Compute a 95 percent confidence interval if we know that both schools have the same unknown standard deviation. The sample standard deviations are 11 and 9 respectively.
   - In the former case, can we accept the hypothesis that the true means of both schools are the same?

Some useful data:

- $X \sim \mathcal{N}(0.1)$, then $P(X > 1.96) = 0.975$, $P(X > 1.64) = 0.95$.
- If T_n is a Student's t with n degrees of freedom, then $P(T_9 > 1.83) = 0.95$, $P(T_9 > 2.26) = 0.975$, $P(T_{79} > 1.66) = 0.95$, $P(T_{79} > 1.99) = 0.975$.