UPPSALA UNIVERSITET

Written examination
Applied mathematics 1MA060
2014–10–21

Allowed aids: writing material. For the grades 3, 4 and 5 one should obtain at least 18, 25 and 32 points, respectively. Solutions should be accompained with explanatory text.

1. *(6p)* Consider the initial value problem

$$\begin{cases} y' + \sin(\varepsilon)y = 0 \\ y(0) = 1 \end{cases}$$

Compute the first three terms of the Taylor expansion

$$y(t;\varepsilon) = y_0(t) + \varepsilon y_1(t) + \varepsilon^2 y_2(t) + h.o.t.,$$

centered at $\varepsilon = 0$.

2. *(6p)* Consider the equation

$$\sin(\varepsilon x^4 + x - 1) = 0.$$

When $\varepsilon = 0$, $x_0 = 1$ is a solution. Compute the first three terms of the Taylor expansion

$$x(\varepsilon) = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + h.o.t.,$$

centered at $\varepsilon = 0$, with $x(0) = 1$.

3. *(6p)* Determine all equilibrium points (a.k.a. stationary solutions) of the system

$$\begin{cases} x' = x^2 + y \\ y' = 2x \end{cases}$$

Determine if they are asymptotically stable.

4. *(6p)* Find all solutions of the PDE

$$u_{tx} = 1$$

that satisfy $u(x,0) = \cos(x)$ and $u(0,t) = \sin(t) + 1$.

5. *(6p)* Find all extremal solutions of the functional

$$\mathcal{J}(y) = \int_0^1 (y(x) + y'(x))^2 + y(x)^2 dx,$$

where y is twice-differentiable and subject to the boundary conditions $y(0) = 0$ and $y(1) = 0$. Is any of these extremals a global minimum? *(A global minimum y_* is a function such that for all other functions y, $\mathcal{J}(y) \geq \mathcal{J}(y_*)$).*

6. *(5p)* Solve the Volterra equation

$$u(y) = 1 + 5\int_0^y u(s)ds,\ 0 \leq y \leq 1.$$

7. *(5p)* Consider the Fredholm equation

$$u(y) = 1 + \varepsilon\int_0^1 u(s)ds,\ 0 \leq y \leq 1.$$

When $\varepsilon = 0$, a solution is $u_0(y) = 1$. Find the first three terms of the Taylor expansion

$$u(y;\varepsilon) = u_0(y) + \varepsilon u_1(y) + \varepsilon^2 u_2(y) + h.o.t.$$

of the solution.

May the Math be with you.