TAKEHOME FINAL, DUE JAN 1, 2017

1MA217, Dynamical Systems

Name: ____

Problem 1, 8 points

1) Take any perturbation of the quadratic family, e.g. a family

$$f_\mu(x) = \mu x(1-x) + \epsilon \cdot \text{ analytic function,}$$

ϵ - small, or another one, and compute the bifurcation diagram for it, i.e. locations of the stable attractors as a function of the parameter.

Make sure that the new family is unimodal - has a single critical point, and is a dynamical system: maps an interval into an interval.

2) As for the quadratic family, compute the accumulation rate of the bifurcation parameters, and the accumulation rate of the distances between two neighboring points in a superstable periodic orbit (superstable=the critical point is in the periodic orbit).

3) Compare with those values for the quadratic family itself.

Problem 2, 6 points
Recall the construction in the Denjoy example of a non-transitive C^1-diffeo: the closed intervals I_k where placed along the orbit $\{R^k_\rho(x)\}$ in the circle of a point x under the rigid rotation R_ρ, so that the distance between interval I_m and I_n is equal to

$$\left(1 - \sum_{n\in\mathbb{Z}} l_n\right) d(x_m, x_n) + \sum_{x_k \in (x_m, x_n)} l_k.$$

In class we have shown that $|f(I_k)| = |I_{k+1}|$ (and we have NOT shown that $f(I_k) = I_{k+1}$), $I_k = [a_k, b_k]$. Show, by induction, that $f(a_k) = a_{k+1}$, i.e., that $f(I_k) = I_{k+1}$ indeed.

Problem 3, 6 points
Let f have an irrational rotation number

$$\rho(f) = [a_1, a_2, \ldots],$$

and be topologically conjugate to the rigid rotation R_f. Set

$$\frac{p_n}{q_n} = [a_1, a_2, a_3, \ldots, a_n].$$

Show that q_n's are such that there exist no k, with $|k| < q_n$, such that

$$f^k(x) \in (x, f^{q_n}),$$

where the last is understood as the arc in the circle with the shortest length.

Problem 4, 7 points

1) Show that the the closure of the set of the recurrent points is in the non-wondering set: $\overline{R(f)} \subset NW(f)$.

2) Give an example of a dynamical system (discrete or continuous) where $NW(f) \not\subset \overline{R(f)}$.

Problem 5, 6 points

Complete the proof of the regularity of the stable manifold - existence of a tangent vector to the curve γ_* (5) on page 17 in my notes).

Problem 6, 5 points

Proof that the correspondence between the points in an N-component horseshoe Λ and the space of bi-infinite sequences Σ_N is a homeomorphism.

Problem 7, 6 points

Prove that (6.14) and (6.15) on page 27 in the lecture notes define a equivariant (that is $Dg(x)E^+(x) = E^+(g(x))$, and similarly for E^-) linear subspaces. (This is essentially done in Lemma 6.2.12 in KH, you may carefully go through the proof and adapt it to our case).

Problem 8, 5 points

Prove that a circle rotation does not have the shadowing property.

Problem 9, 6 points Suppose a continuous $f : [0,1] \mapsto [0,1]$ has a periodic orbit $\{x_1 < x_2 < x_3 < x_4\}$ such that $f(x_i) = x_{i+1}$ for $i < 4$ and $f(x_4) = x_1$. Show that f has periodic points of all periods.