Slow-fast dynamics in differential equations and maps

Ilya V. Schurov
National Research University Higher School of Economics

Slow-fast systems (i.e. systems with different time scales, slow and fast motions) are studied in the theory of differential equations for a long time. Today it is well-developed branch of mathematics involving sophisticated but elegant techniques, deep results and various applications. The simplest slow-fast system is a family of differential equations of the form

$$\begin{cases} \dot{x} = f(x, y, \varepsilon) \\ \dot{y} = \varepsilon g(x, y, \varepsilon), \end{cases} \tag{1}$$

where ε is a small parameter, y is a slow variable, x is a fast variable. For $\varepsilon = 0$, variable y is a constant and the nullcline of the first equation is a curve consisting of equilibrium points. Segments of this curve consisting of *hyperbolic* points persist (as invariant curve) for small $\varepsilon \neq 0$ and most of the trajectories are attracted to such curves. So their asymptotic properties are crucial for the analysis of the slow-fast system.

One can consider natural counterpart of slow-fast systems in discrete time dynamics: iterations of maps

$$(x, y) \mapsto (x + f(x, y, \varepsilon),\ y + \varepsilon g(x, y, \varepsilon)).$$

No general theory of such systems is known.

In the present talk based on the joint work-in-progress with Yulij S. Ilyashenko we will discuss an approach to study slow-fast maps. This approach allows to embed slow-fast map into a flow of slow-fast system in some special regions and prove theorems on asymptotics of its invariant manifolds similar to such of the slow-fast systems in continuous time.