Slow-fast dynamics in differential equations and maps

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Slow-fast systems (i.e. systems with different time scales, slow and fast motions) are studied in the theory of differential equations for a long time. Today it is well-developed branch of mathematics involving sophisticated but elegant techniques, deep results and various applications. The simplest slow-fast system is a family of differential equations of the form

\[
\begin{aligned}
\dot{x} &= f(x, y, \varepsilon) \\
\dot{y} &= \varepsilon g(x, y, \varepsilon),
\end{aligned}
\]  

where \( \varepsilon \) is a small parameter, \( y \) is a slow variable, \( x \) is a fast variable. For \( \varepsilon = 0 \), variable \( y \) is a constant and the nullcline of the first equation is a curve consisting of equilibrium points. Segments of this curve consisting of hyperbolic points persist (as invariant curve) for small \( \varepsilon \neq 0 \) and most of the trajectories are attracted to such curves. So their asymptotic properties are crucial for the analysis of the slow-fast system.

One can consider natural counterpart of slow-fast systems in discrete time dynamics: iterations of maps

\[ (x, y) \mapsto (x + f(x, y, \varepsilon), y + \varepsilon g(x, y, \varepsilon)). \]

No general theory of such systems is known.

In the present talk based on the joint work-in-progress with Yulij S. Ilyashenko we will discuss an approach to study slow-fast maps. This approach allows to embed slow-fast map into a flow of slow-fast system in some special regions and prove theorems on asymptotics of its invariant manifolds similar to such of the slow-fast systems in continuous time.