Asymptotic laws for some sequential dynamical systems

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Let $(\tau_n)_{n\geq 1}$ be a "sequential" dynamical system, i.e. a sequence of non singular transformations on a probability space (X, m). We consider different examples where, after normalization, a limit theorem can be obtained for the sums $\sum_{k=1}^n f(\tau_k \circ \tau_{k-1} \cdots \circ \tau_1 x)$ when f is a regular function on X.

We will take for (τ_n):
1) a sequence of β-transformations $\tau_n : x \to \beta_n x \bmod 1$, with $\beta_n > 1$;
2) a sequence of toral automorphisms $\tau_n : x \to A_n x \bmod \mathbb{Z}^d$ with $A_n \in \{A, B\}$, where A and B are two matrices in $SL(d, \mathbb{Z})$.

3) The asymptotic behaviour of the sums $\sum_{k=1}^n f(q_k x \bmod 1)$, where (q_n) is an increasing sequence of integers will be also discussed, an old question since A. Zygmund, M. Kac, R. Fortet among others, recently considered again by C. Aistleitner and I. Berkes.

In case 1) the spectral gap method for the corresponding transfer operators and a martingale argument can be used to prove a Central Limit Theorem (joint work with A. Raugi).

In case 2) we give conditions which imply decorrelation and enable to apply the method of "multiplicative systems" of Komlòs, providing a Central Limit Theorem for the sums $\sum_{k=1}^n f(\tau_k \circ \tau_{k-1} \cdots \circ \tau_1 x)$ when f is a regular function on $\mathbb{T}^d$. These conditions hold for 2×2 matrices with positive coefficients. In dimension d, they can be applied when $\tau_n x = A_n(\omega) x \bmod \mathbb{Z}^d$, with independent choice of $A_n(\omega) \in \{A, B\}$, A and B in $SL(d, \mathbb{Z}^+)$, in order to prove a "quenched" CLT (question also recently considered by A. Ayyer, C. Liverani and M. Stenlund).

In case 3) we recall an ancient example where the CLT does not hold, but where a mixture of Gaussian laws is obtained as asymptotic law, and we give a generalization of this fact. The results in 2) and 3) are joint works with S. Le Borgne and M. Roger.