DYNAMICS OF LINEAR OPERATORS IN FINITE AND INFINITE DIMENSIONS

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Abstract

Let X be Banach space. A linear operator $T : X \to X$ is called hypercyclic if there exists a vector $x \in X$ so that the sequence $\{T^n x : n = 0, 1, 2, \ldots\}$ is dense in X. This means that there are vectors whose orbits under T are dense. I will review some recent developments concerning dynamics of linear operators. In particular, we treat a new notion related to linear dynamics, which can be viewed as a "localization" of the notion of hypercyclicity. In particular, let T be a bounded linear operator acting on a Banach space X and let x be a non-zero vector in X such that for every open neighborhood $U \subset X$ of x and every non-empty open set $V \subset X$ there exists a positive integer n such that $T^n U \cap V \neq \emptyset$. In this case T will be called a J-class operator. We investigate the class of operators satisfying the above property and provide various examples. It is worthwhile to mention that many results from the theory of hypercyclic operators have their analogues in this setting. For example we establish results related to the Bourdon-Feldman theorem and we characterize the J-class weighted shifts. We would also like to stress that even non-separable Banach spaces which do not support topologically transitive operators, as for example $l^\infty(\mathbb{N})$, do admit J-class operators. We also extend the notion of a J-class operator to that of a J-class tuple of operators. We then show that the class of hypercyclic tuples of operators forms a proper subclass to that of J-class tuples of operators. What is rather remarkable is that in every finite dimensional vector space over $\mathbb{R}$ or $\mathbb{C}$, a pair of commuting matrices exists which forms a J-class non-hypercyclic tuple. This comes in direct contrast to the case of hypercyclic tuples where the minimal number of matrices required for hypercyclicity is related to the dimension of the vector space. In this direction we prove that the minimal number of diagonal matrices required to form a hypercyclic tuple on $\mathbb{R}^n$ is $n + 1$, thus complementing a recent result due to Feldman. The previous results are joint works with D. Hadjiloucas and A. Manoussos.

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