

On the iterates of the Euler function

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Let $\phi(n)$ be the Euler function of the positive integer n . For a positive integer k , let $\phi^{(k)}(n)$ be the k th fold iterate of the function $\phi(n)$. In my talk, I will look at the range of the function $\phi^{(k)}(n)$. For example, putting $V_k(x) = \#\{\phi^{(k)}(n) \leq x\}$, then for x sufficiently large the estimate

$$\#V_k(x) \leq \frac{x}{(\log x)^k} \exp(13k^{3/2}(\log \log x \log \log \log x)^{1/2})$$

holds uniformly in $k \geq 1$. Under the prime k -tuples conjecture I show that $\#V_k(x) \gg_k x/(\log x)^k$. I will also give the main ideas of an unconditional proof of this lower bound when $k = 2$. These results have been obtained jointly with Carl Pomerance.