

UPPSALA UNIVERSITET  
 Matematiska institutionen  
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Exercises 1  
 Logik II  
 2008

*Deadline:* 2008-10-14 (at 10.00)  
*Hjälpmedel:* Course material or any book.  
*Maximal poäng:* None  
*Instruktioner:* See below.

Submit at least neatly handwritten (and individually prepared) solutions of the following problems. You may use any book as help, but state and refer clearly to which reference (and what part of that reference you use) if you do so. All numbered references below are to chapter I of the course literature unless otherwise stated. It is a part of all problems to decide how detailed solution you should hand in. (A guideline is to think that the solution should be clear to all participants of the course.) Ask me if you have any questions!

Hand in the problems to me personally or in my mailbox before deadline. No solutions handed in after the deadline will be considered unless you have an agreement with me.

1. Describe the details and the reason for the definition in section 3 of the notions of *prime formulas*, *propositional formulas*, *propositional valuations*, *satisfiable*, *tautologies* and *tautologically implies*.
2. Work through the details of Case 1 in the proof of Theorem 5.12.
3. Prove Theorem 4.27.
4. Solve Exercise 18.
5. Prove that if  $|L| = \kappa$  for an infinite cardinal  $\kappa$  then  $\text{FORM}_L = \kappa$ .
6. Let  $L_G$  be the language of groups and let  $\kappa$  be an infinite cardinal. Show that the class of groups of cardinality  $\kappa$  is not first order definable. That is, show that there is no theory  $T$  such that  $\mathcal{A} \models T \Leftrightarrow |\mathcal{A}| = \kappa$  and  $\mathcal{A}$  is a group.
7. Work through (some of) the (interesting?) details of the proof of the Main consistency theorem, in particular Lemmata 5.28 – 33, in the uncountable case.
8. In his travels, Gulliver did not visit the *world of infinite countries and finite logic*. If he had, he would have discovered that the inhabitants of that world live in an infinite number of countries, but they only believe in finiteness in the sense that they restrict their definition of model in logic to only structures with a finite universe.
  - (a) Suppose that Gulliver were to draw a map of that world. How many colours did he have to use to colour each pair of neighbouring countries with different colours? Two countries are neighbours if and only if they have a common border, not counting borders of length 0. (Hint: You might want to have a look at the (first) computer-aided proof in mathematics, by Appel and Haken from 1976.)
  - (b) Is there any important result of our course that fail to hold if we only have finite models?