UPPSALA UNIVERSITET
Matematiska institutionen
Göran Hamrin

Exercises 4
Logik II
2008

Deadline:	2008-12-02 (at 10.00)
Hjälpmedel:	Course material or any book.
Maximal poäng:	None
Instruktioner:	See below.

Submit at least neatly handwritten (and individually prepared) solutions of the following problems. You may use any book as help, but state and refer clearly to which reference (and what part of that reference you use) if you do so. All numbered references below are to chapter I of the course literature unless otherwise stated. It is a part of all problems to decide how detailed solution you should hand in. (A guideline is to think that the solution should be clear to all participants of the course.) Ask me if you have any questions!

Hand in the problems to me personally or in my mailbox before deadline. No solutions handed in after the deadline will be considered unless you have an agreement with me.

1. Let $F : \mathbb{N}^2 \to \mathbb{N}$ be total recursive and define $G_m : \mathbb{N} \to \mathbb{N}$ by $G_m(y) = F(m, y)$, for all $m \in \mathbb{N}$. Construct a total recursive $H : \mathbb{N} \to \mathbb{N}$ such that $H \neq G_m$, for all $m \in \mathbb{N}$.

2. Let $F : \mathbb{N} \to \mathbb{N}$ be a partial function and let $m \in \mathbb{N}$. Construct $G : \mathbb{N} \to \mathbb{N}$ such that G is *not* recursive and such that $G(x) \simeq F(x)$ for all $x \leq m$.

3. Let $c \in \mathbb{N}$ and show that $c \in W_x$ is undecidable.

4. Show that $\emptyset = W_x$ is undecidable.

5. Let $f : \mathbb{N} \to \mathbb{N}$ be total recursive and show that for each m there is $n \geq m$ such that $\varphi_n = \varphi_{f(n)}$.

6. Show that there is $n \in \mathbb{N}$ such that $W_n = \{n, 2n, 3n, \ldots\}$, such that $E_n = \{\varphi_n(x) : x \in W_n\}$ and such that $W_n = E_n$.

7. Let $A \subseteq \mathbb{N}$ be infinite and show that A is recursive if and only if A is the image of a total recursive and *strictly increasing* function.

8. A and B are *recursively inseperable* if it does not exist a recursive C such that $A \subseteq C$ and $B \subseteq \mathcal{C}(C)$, where $\mathcal{C}(X)$ is the set-theoretic complement of X with respect to $\mathbb{N}$. Show that $K_0 = \{x : \varphi_x(x) = 0\}$ and $K_1 = \{x : \varphi_x(x) = 1\}$ are recursively inseperable.