UPPSALA UNIVERSITET
Matematiska institutionen
Göran Hamrin

Exercises 3
Logik II
2008

Deadline:	2008-11-25 (at 10.00)
Hjälpmedel:	Course material or any book.
Maximal poäng:	None
Instruktioner:	See below.

Submit at least neatly handwritten (and individually prepared) solutions of the following problems. You may use any book as help, but state and refer clearly to which reference (and what part of that reference you use) if you do so. All numbered references below are to chapter I of the course literature unless otherwise stated. It is a part of all problems to decide how detailed solution you should hand in. (A guideline is to think that the solution should be clear to all participants of the course.) Ask me if you have any questions!

Hand in the problems to me personally or in my mailbox before deadline. No solutions handed in after the deadline will be considered unless you have an agreement with me.

1. Show that the complement of a recursive relation $P \subseteq \mathbb{N}^n$ is recursive.

2. Let P_1 and P_2 be disjoint recursive subsets of $\mathbb{N}$ and let G_5, G_7 and G_{11} be total (partial) recursive functions. Define $F : \mathbb{N} \to \mathbb{N}$ by

$$F(x) = \begin{cases} G_5(x) & \text{if } P_1(x) \\ G_7(x) & \text{if } P_2(x) \\ G_{11}(x) & \text{otherwise.} \end{cases}$$

   Show that F is total (partial) recursive.

3. Let $G : \mathbb{N}^n \to \mathbb{N}$ and $H : \mathbb{N}^{n+2} \to \mathbb{N}$ be total recursive functions. Define $F : \mathbb{N}^{n+1} \to \mathbb{N}$ by

$$F(\bar{x}) = \begin{cases} F(0, \bar{x}) & = & G(\bar{x}) \\ F(k+1, \bar{x}) & = & H(F(k, \bar{x}), k, \bar{x}) \end{cases}$$

   Show that F is total recursive.

4. Use Gödel's β-function to show that $F(n) = n!$ is total recursive.

5. Show that there is a function F, defining the Fibonacci numbers, such that F is total recursive.

6. Let $\Pi(x, y) : \mathbb{N}^2 \to \mathbb{N}$ be defined by $\Pi(x, y) = 2^x(2y + 1) - 1$. Show that Π is a bijective and total recursive function. Show also that the projections Π_0, Π_1, defined via $\Pi(\Pi_0(z), \Pi_1(z)) = z$ are total recursive.

7. Let $I \subseteq \mathbb{R}$ be an interval. Show that if $f : I \to \mathbb{R}$ is an uniformly continuous function then $(\forall x, y \in I^*)(x \simeq y \Rightarrow f(x) \simeq f(y))$. ($x \simeq y$ if $x - y$ is infinitesimal, as usual. I^* is the interpretation of I in (a non-standard model) $\mathbb{R}^*$.)

8. Let $C \subseteq \mathbb{R}$ be a closed and bounded interval. Use non-standard analysis to show that if $f : C \to \mathbb{R}$ is a continuous function on C then f is uniformly continuous on C.