Comments on 2283 Exam #2   5/09

Answers for problems 1, 2, 4. Then a remark about #3.

$\boxed{1}$

(a) conv — root test

(b) conv. — limit comp.; $p = 2$

(c) conv. — root test

(d) conv. — limit comp; then $p = 1.5$

(e) conv — compare to n^{-2}

(f) conv. — begin with limit comp.; then integral test and $\ln t < t^{\varepsilon}$ for $t \geq t_{\varepsilon}$

$\boxed{2}$

(a) div   (b) cond conv   (c) cond conv

(d) div — use ratio test $|a_n| \not\to 0$ (e) abs conv.

$\boxed{4}$

(a) $-\frac{1}{4} \leq x \leq \frac{1}{4}$

(b) none — power series conv absolutely on $|x| < r^*$, r^* = radius of conv. Only $x = \pm r^*$ can be points of cond. conv.

(c) $x = -\frac{1}{4} \Rightarrow$ series $= 8 \sum_{n=3}^{\infty} \frac{(-1)^n}{n^2-1}$

$$= 4 \sum_{n=3}^{\infty} (-1)^n \left[\frac{1}{n-1} - \frac{1}{n+1} \right]$$

$$= 4(-1)\left[\frac{1}{2} - \frac{1}{4} + \frac{1}{4} - \frac{1}{6} + \frac{1}{6} - \frac{1}{8} \pm \cdots \right]$$

$$+ 4(+1)\left[\frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \frac{1}{7} - \frac{1}{9} \pm \cdots \right]$$

$$= -4\left(\frac{1}{2}\right) + 4\left(\frac{1}{3}\right) = -2 + \frac{4}{3} = -\frac{2}{3}$$

$\boxed{3}$ Remember that

$$\left| \sum_{n=N+1}^{\infty} c_n \right| \leqq \sum_{n=N+1}^{\infty} |c_n| \qquad \text{and} \qquad \sum_{n=N+1}^{\infty} g(n) \leqq \int_N^{\infty} g(x)\,dx$$

for $g > 0$, g decreasing.

<u>Want</u>

$$\left| \sum_{n=N+1}^{\infty} \sin(n) \frac{n + 8\sqrt{n+40}}{n^5+1} \right| < 10^{-11}.$$

It is enough to make:

$$\sum_{N+1}^{\infty} |\sin(n)| \frac{n + 8\sqrt{n+40}}{n^5+1} < 10^{-11}$$

or $$\sum_{N+1}^{\infty} \frac{n + 8\sqrt{n+40}}{n^5+1} < 10^{-11}.$$

<u>Keep</u> $N \geqq 10$ say. Then $\sqrt{n+40} < n$ since $\sqrt{\frac{1}{n} + \frac{40}{n^2}} < 1$.

Enough to make:

$$\sum_{N+1}^{\infty} \frac{n + 8n}{n^5} < 10^{-11}.$$

Or,

$$\sum_{N+1}^{\infty} \frac{10n}{n^5} < 10^{-11}.$$

So, it's enough to make

$$10 \int_N^{\infty} x^{-4}\,dx < 10^{-11} \qquad \text{or} \qquad \frac{1}{3N^3} < 10^{-12}.$$

Ie: $3N^3 > 10^{12}$, or $N > \left(\frac{1}{3}\right)^{\frac{1}{3}} \times 10^4$. $\left(\sqrt[3]{\tfrac{1}{3}} < 1\right)$

$N \geqq 10^4$ certainly suffices.