$\boxed{2010\text{-}03\text{-}12}$

(1)

a) $$\frac{x-\sin x}{x^3} = \frac{x-\left(x-\frac{x^3}{3!}+\mathcal{O}(x^4)\right)}{x^3} = \frac{1}{3!}+\mathcal{O}(x) \longrightarrow 1/6$$

b) $$\sqrt{x+1}-\sqrt{x-1} = \frac{(\sqrt{x+1}-\sqrt{x-1})(\sqrt{x+1}+\sqrt{x-1})}{\sqrt{x+1}+\sqrt{x-1}} =$$

$$= \frac{2}{\sqrt{x+1}+\sqrt{x-1}} = \frac{2}{\sqrt{x}}\cdot\frac{1}{\sqrt{1+1/x}+\sqrt{1-1/x}} \rightarrow \boxed{0}$$

(2) $$2x^3y-y^3x=1 \Rightarrow 6x^2y+2x^3y'-3y^2y'x-y^3=0$$

$$\Rightarrow y' = \frac{6x^2y-y^3}{3y^2x-2x^3} \Rightarrow \frac{dy}{dx}(1,1) = \frac{6-1}{3-2} = \underline{\underline{5}}.$$

$y-1=5(x-1)$

Svar: $\boxed{y=5x-4}$

(3) a)

$$\frac{1}{x^2(x+1)} = \frac{A}{x^2}+\frac{B}{x}+\frac{C}{x+1}$$

Handpål: $A=1$, $B=?$ $C=1$

$$\frac{1}{x^2(x+1)}-\frac{1}{x^2}-\frac{1}{x+1} = \frac{1-(x+1)-x^2}{x^2(x+1)} = \frac{-x(x+1)}{x^2(x+1)} = -\frac{1}{x} \qquad B=-1$$

$$= \frac{1}{x^2}-\frac{1}{x}+\frac{1}{x+1} \text{ ger}$$

$$\left[-\frac{1}{x}-\ln|x|+\ln|x+1|\right]_1^\infty = 1+\ln\left|\frac{\infty+1}{\infty}\right|+\ln 1-\ln 2 = \boxed{1-\ln 2} \qquad \left(\ln\frac{e}{2}\right)$$

(4) a) $r^2-r=0 \quad r=0,1 \quad y_H=\underline{A+Be^x}$

$y_p=Ae^x\cdot x \quad y_p'=Ae^x+Axe^x \quad y_p''=2Ae^x+Axe^x$

$(2Ae^x+Axe^x)-(Ae^x+Axe^x)=e^x \Leftrightarrow Ae^x=e^x \Rightarrow \underline{A=1}$

$\boxed{y=A+Be^x+xe^x}$

$y(0)=A+B=0 \Rightarrow A=1$

$y'(0)=B+1=0 \Rightarrow B=-1$

Svar: $\boxed{y=1-e^x+xe^x}$

4b)

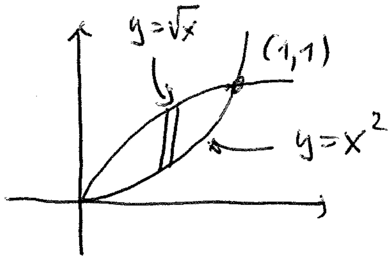
$$\int \frac{\sqrt{x}}{\sqrt{x}-1}\,dx = \left\{ \begin{array}{l} \sqrt{x}=t \\ x=t^2 \\ dx=2t\,dt \end{array}\right\} = \int \frac{t\cdot 2t\,dt}{t-1} =$$

$$= \int \frac{2t^2}{t-1}\,dt = \int \frac{2t^2-2t+2t-2+2}{t-1}\,dt = \left\{\begin{array}{l}\underline{\text{Alternativ:}}\\ \text{polynom-}\\ \text{division!}\end{array}\right\}$$

$$= \int \frac{2t(t-1)+2(t-1)+2}{t-1}\,dt = \int \left(2t+2+\frac{2}{t-1}\right)dt =$$

$$= \left[t^2+2t+2\ln|t-1|\right] = \boxed{x+2\sqrt{x}+\ln(\sqrt{x}-1)^2+C}$$

⑤ Rörformeln ger:




$$V=\int_0^1 2\pi x\left(\sqrt{x}-x^2\right)dx =$$

$$= 2\pi\int_0^1 \left(x^{3/2}-x^3\right)dx = 2\pi\left[\frac{x^{5/2}}{5/2}-\frac{x^4}{4}\right]_0^1 = 2\pi\left(\frac{2}{5}-\frac{1}{4}\right) =$$

$$= 2\pi\left(\frac{8}{20}-\frac{5}{20}\right) = \boxed{\frac{3\pi}{10}\ \text{V.e}}$$

⑥ $y=\dfrac{2x^3}{x^2-1}=\dfrac{2x^3}{(x-1)(x+1)}$ definierad för $x\neq\pm 1$

<u>Lodräta asymptoter</u> blir: $x=1$ och $x=-1$.

Vi "<u>ser</u>":

$$\left\{\begin{array}{l} x\to 1^+ \Rightarrow y\to+\infty \\ x\to 1^- \Rightarrow y\to-\infty \\ x\to -1^+ \Rightarrow y\to+\infty \\ x\to -1^- \Rightarrow y\to-\infty \end{array}\right.$$

<u>Sned asymptot</u>: $y = \dfrac{2x^3}{x^2-1} = \dfrac{2x(x^2-1)+2x}{x^2-1} =$

$= 2x + \dfrac{2x}{x^2-1} \Rightarrow y = 2x$ sned asymptot i $\pm\infty$

och vi närmar oss <u>ovanifrån</u> i $+\infty$ $\left(\dfrac{2x}{x^2-1} > 0\right)$

<u>underifrån</u> i $-\infty$ $\left(\dfrac{2x}{x^2-1} < 0\right)$

<u>Extremvärden</u>. $y' = \dfrac{(x^2-1)6x^2 - 2x^3 \cdot 2x}{(x^2-1)^2} = \dfrac{2x^4-6x^2}{(x^2-1)^2} =$

$= \dfrac{2x^2(x^2-3)}{(x^2-1)^2} = \dfrac{2x^2(x-\sqrt{3})(x+\sqrt{3})}{(x^2-1)^2}$

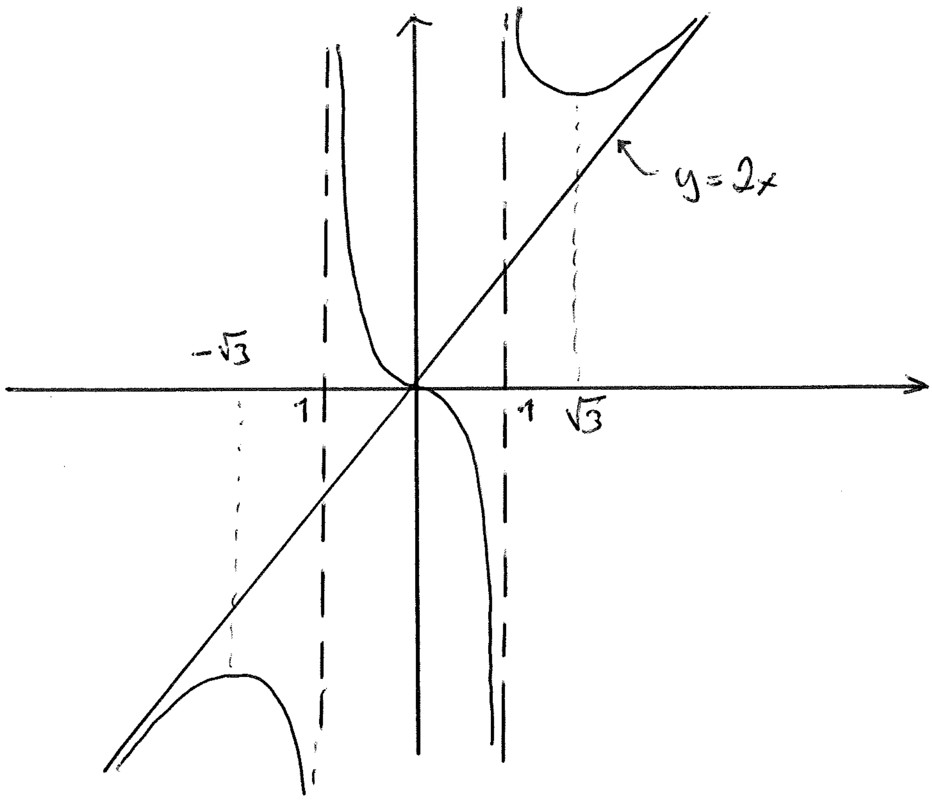
<u>Kritiska punkten</u>: $x = 0$, $x = \pm\sqrt{3}$

| x: | | $-\sqrt{3}$ | | $-1$ | | $0$ | | $1$ | | $\sqrt{3}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y'$ | + | 0 | − | ≀ | − | 0 | − | ≀ | − | 0 | + |
| $y$ | ↗ | LOKALT MAX | ↘ | | ↘ | | ↘ | | ↘ | LOKALT MIN. | ↗ |

och $x = \sqrt{3} \Rightarrow y = \dfrac{6\sqrt{3}}{2} = 3\sqrt{3}$

$x = -\sqrt{3} \Rightarrow y = -3\sqrt{3}$

<u>OBS</u>: y är <u>UDDA</u>!



7. a) $\sum_{n=1}^{\infty} \frac{n^2\sqrt{n}}{(n^2+1)^2}$

$\frac{n^2\sqrt{n}}{(n^2+1)^2} \approx \frac{n^{5/2}}{n^4} = \frac{1}{n^{4-5/2}} = \frac{1}{n^{3/2}}$ Jämför med denna!

$$\frac{\frac{n^2\sqrt{n}}{(n^2+1)^2}}{\frac{1}{n^{3/2}}} = \frac{n^{3/2} \cdot n^2 \cdot n^{1/2}}{(n^2+1)^2} =$$

$$= \frac{n^4}{n^4+2n^2+1} = \frac{1}{1+\frac{2}{n^2}+\frac{1}{n^4}} \underset{n\to\infty}{\longrightarrow} 1$$

Vi vet att $\sum_1^{\infty} \frac{1}{n^{3/2}}$ är <u>konvergent</u> (p-serie med $p = 3/2 > 1$).

Enl. jämförelsesatsen är då också den givna serien <u>konvergent</u>!

b) $\sum_{n=1}^{\infty} \frac{n}{\ln(5+e^{n^2})}$

$\ln(5+e^{n^2}) \approx \ln(e^{n^2}) = n^2$

så termerna $\approx \frac{n}{n^2} = 1/n$

( <u>Alt. 1.</u> $\frac{n}{\ln(5+e^{n^2})} > \frac{n}{\ln(e^{n^2}+e^{n^2})} = \frac{n}{\ln(2e^{n^2})} = \frac{n}{\ln 2 + n^2} > \frac{n}{n^2+n^2} = \frac{1}{2n}$ Divergent )

<u>Alt 2.</u> Jämför med $\sum \frac{1}{n}$: ← <u>Divergent!</u>

$$\frac{\frac{n}{\ln(5+e^{n^2})}}{\frac{1}{n}} = \frac{n^2}{\ln\left(e^{n^2}\left(\frac{5}{e^{n^2}}+1\right)\right)} = \frac{n^2}{\ln(e^{n^2})+\ln\left(1+\frac{5}{e^{n^2}}\right)} =$$

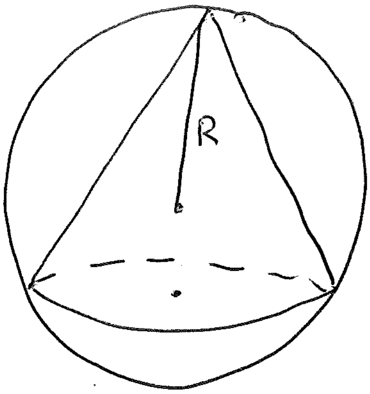
$$= \frac{n^2}{n^2+\ln(1+\underbrace{5/e^{n^2}}_{\to 0})} \longrightarrow 1$$

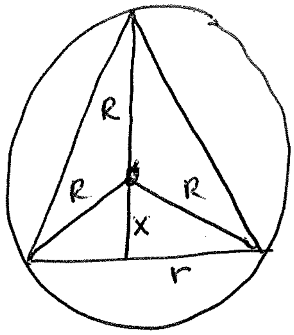
<u>SLUTSATS!</u>

<u>DIVERGENT.</u>

8.

$$V(\text{kon}) = \frac{\text{HÖJD} \cdot \text{BASAREAN}}{3}$$

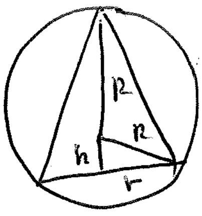



$$\text{Höjden} = R + x \underset{\substack{\uparrow \\ \text{Pythagoras}}}{=} R + \sqrt{R^2 - r^2} \ \ldots$$

eller

(lättare!)




$r = \sqrt{R^2 - h^2} \Rightarrow$ Basarean blir

$\pi r^2 = \pi(R^2 - h^2)$

Volymen som funktion av h

blir

$\boxed{(-R \leq h \leq R)}$

$$V(h) = \frac{h\pi(R^2 - h^2)}{3} \qquad 0 \leq h \leq R$$

$$V'(h) = \frac{\pi}{3}\left[(R^2 - h^2) + h \cdot (-2h)\right] =$$

$V(-R) = V(R) = 0$

$$= \frac{\pi}{3}(R^2 - 3h^2)$$

$$V'(h) = 0 \text{ om } \boxed{h = \frac{R}{\sqrt{3}}}$$

Maximal volym: $V = \dfrac{\frac{R}{\sqrt{3}}\pi\left(R^2 - \frac{R^2}{3}\right)}{3} =$

$$= \boxed{\frac{2\pi R^3}{9\sqrt{3}}}$$

— X —