UPPSALA UNIVERSITET | ÖVNINGAR, Blad 2

Matematiska institutionen
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Teori för differentialekvationer
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2.1. Find all solutions to the following equations:

(a)
$$y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = 0;$$

(b)
$$x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0;$$

(c)
$$y\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0;$$

(d)
$$y\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u;$$

(e)
$$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 0;$$

(f)
$$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = x;$$

(g)
$$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = x + y.$$

2.2. Solve the Cauchy problem

$$\frac{\partial u}{\partial x} + (x+y)\frac{\partial u}{\partial y} = 1,$$
$$u(x, -x) = 0.$$

2.3. Solve the Cauchy problem

$$(1+x^2)\frac{\partial u}{\partial x} + 2xy\frac{\partial u}{\partial y} = 0,$$
$$u(x, x+x^3) = h(x).$$

2.4. Find a function u defined in some open neighborhood of the x-axis in $\mathbf{R}^2$ such that

$$x^2\frac{\partial u}{\partial x} + (y+1)\frac{\partial u}{\partial y} = 0,$$
$$u(x, 0) = x.$$

Prove that if u is a solution to the differential equation in the whole plane, then $u(x,0)$ is constant for $x \geqslant 0$. By way of contrast, $u(x,0)$ need not to be constant for $x < 0$, but the limit $\lim_{x\to-\infty} u(x,0)$ exists.