UPPSALA UNIVERSITET
Matematiska institutionen
Christer Kiselman

ÖVNINGAR, Blad 4
Teori för differentialekvationer
1998 01 28

4.1. Solve the initial-value problem

$$\frac{\partial u}{\partial x} = \left(\frac{\partial u}{\partial y}\right)^2, \qquad u(0,y) = y^2/2,$$

using the method of envelopes as well as the method of characteristic strips. Compare the two methods of computing.

4.2. Solve the Cauchy problem

$$\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} = 1, \qquad u(x,-x) = 1.$$

How many solutions are there?

4.3. Solve the initial-value problem

$$\left(\frac{\partial u}{\partial x}\right)^3 + \frac{\partial u}{\partial y} = u, \qquad u(s,0) = s.$$

4.4. Solve the initial-value problem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} = u, \qquad u(s,0) = s^2.$$

Try both characteristic strips and envelopes of affine solutions. Which method is the easiest in this case?

4.5. Solve the Cauchy problem

$$\left(\frac{\partial u}{\partial x}\right)^2 + 4y\left(\frac{\partial u}{\partial y}\right)^2 = 2, \qquad u(s,1) = s+1.$$

How many solutions are there?

4.6. Find the solution to the equation

$$x\left(\frac{\partial u}{\partial x}\right)^2 + y\frac{\partial u}{\partial y} = 0$$

which satisfies $u(s,1) = -s$.

4.7. Solve the Cauchy problem

$$x\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^3 = 1, \qquad u(s,0) = \sqrt{s}, \quad s > 0.$$