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ÖVNINGAR, Blad 5
Teori för differentialekvationer
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5.1. Determine all characteristic curves to the equation

$$\frac{\partial^2 u}{\partial x^2} - y^2 \frac{\partial^2 u}{\partial y^2} = 0.$$

Transform the equation to normal form in the open set $y \neq 0$.

5.2. Determine the characteristic curves to the equation

$$\frac{\partial^2 u}{\partial x^2} - 9x^4 \frac{\partial^2 u}{\partial y^2} - 6xu = 0.$$

Transform the equation to normal form in the domain $x > 0$.

5.3. Determine the characteristic curves to the equation

$$x^2 \frac{\partial^2 u}{\partial x^2} - 2x \frac{\partial^2 u}{\partial x \partial y} + \frac{3}{4} \frac{\partial^2 u}{\partial y^2} + \frac{1}{2} \frac{\partial u}{\partial y} = 0.$$

Transform the equation to normal form in all of $\mathbf{R}^2$. Find the general solution.

5.4. Solve the differential equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$$

with the initial-value conditions $u(x,0) = x$, $u_t(x,0) = 0$. *Hint:* Experiment with exponential functions multiplied by solutions to the ordinary wave equation.

5.5. Solve the equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0$$

in the domain $t > 0$, $x > 0$ with the boundary conditions

$$u(x,0) = x^2, \qquad \frac{\partial u}{\partial t}(x,0) = 0, \qquad u(0,t) = 0.$$

Hint: Think about even and odd functions.

5.6. Solve the initial-value problem

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \frac{\partial^2 u}{\partial x_3^2} \right)$$

with the conditions

$$u(x,0) = 0,$$
$$\frac{\partial u}{\partial t}(x,0) = \begin{cases} 1, & |x| \leqslant a \\ 0, & |x| > a. \end{cases}$$

Discuss the behavior of the solution at the point $(0, a/c)$; it is not continuous there, a fact which depends on the discontinuities in the initial conditions on the sphere $t = 0$, $|x| = a$. The phenomenon is called the focusing effect. *Hint:* Show first that u is a function of $(|x|, t) = (r, t)$ and that $v = ru$ solves the wave equation in the variables (r, t). (This is special for three space variables.)