

# Pierre Lelong 1912—2011

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Pierre Lelong died in Paris on October 12, 2011. He was born on March 14, 1912, also in Paris. His mother and father lived long lives too. Closest to him are his wife France Lelong, four children with his first wife Jacqueline Lelong-Ferrand, and grandchildren. His family was, as he once told me, by tradition atheists for several generations. Like his mother, he felt profoundly Alsatian, but his father was a true Parisian boy with ancestors from the Massif central (France Lelong, personal communication 2012-11-06, 2012-11-07).

Among mathematicians Lelong was best known as a pioneer in the theory of several complex variables and above all as the one who had introduced the plurisubharmonic functions and wrote extensively about them. But in French politics he was well known and highly respected as Charles de Gaulle's advisor during two important years just at the beginning of the Fifth Republic. This was clear for instance when he came to Sweden to sit on the jury for Leif Abrahamsson's PhD Thesis presentation in 1982, and had his flight ticket paid by the French Ministry of Foreign Affairs. For this reason I was in contact with French diplomats in Stockholm, who incredulously wondered how I could have such an important guest.

Lelong was elected a corresponding member of the French Academy of Sciences in 1980 and as a member in 1985. That was of course a most important recognition. However, it came rather late in his life. He had several orders, among them *La Légion d'honneur* (Legion of Honor), where he received the degree of *chevalier* (Knight) in 1959, *officier* (Officer) in 1967, and finally *commandeur* (Commander).

## Lelong's mathematics

From Lelong's very large production during more than sixty years, 1937–1999, I can mention just a small part. In *MathSciNet* he is the author of 108 articles, to which one should add one from 1937 and one from 1938 which are not mentioned there; if we add *Related Publications* there are a total of 129 hits. He has made several important contributions to science. Among these I consider the most significant to be the introduction of the class of plurisubharmonic functions (1942); the Lelong number (1950); closed positive currents and integration on an analytic set (1957a, 1957b). But these are far from the only ones.

## Plurisubharmonic functions

The class of functions which are now known as plurisubharmonic was introduced by Kiyoshi Oka (1942) and Pierre Lelong (1942), who worked independently of each other in Japan and France, respectively. As a matter of fact, Oka did his research already in 1935 (Toshio Nishino, personal communication 1997-10-03) at the University of Hiroshima, where he was Assistant Professor during the period 1932–1938, while Lelong was *Chargé de conférences* in Mechanics in Paris. As Lelong once told me, the two never met.

Oka used the term *fonctions pseudoconvexes* ‘pseudoconvex functions’—they prefer to live in pseudoconvex domains—while Lelong coined the term now in use, *fonctions plurisousharmoniques* ‘plurisubharmonic functions’, to emphasize their relation to subharmonic functions: they are subharmonic in many ways. And that is clear from the definition: a function defined in an open set in the space of several complex variables is defined to be *plurisubharmonic* if its restriction to every complex line is subharmonic, and in addition it is upper semicontinuous (the latter requirement is the same as for subharmonic functions).

If  $h$  is a holomorphic function, then  $\log|h|$  is plurisubharmonic, and this is a first important relation to the holomorphic functions. But the plurisubharmonic functions are easier to manipulate: for instance, the maximum of two plurisubharmonic functions is in the same class—no similar conclusion is allowed for the holomorphic functions. This property shows that these functions are, as Lelong put it, *souples* ‘supple’ compared with the classical objects, which are rigid and more difficult to do carpentry on. In an article (1994) Lelong describes these *objets souples* and how they have been developed during the twentieth century. The plurisubharmonic functions are the archetype for these supple objects. They resemble in some respects the convex functions; in others the subharmonic.

An important property is that the class is invariant under holomorphic coordinate changes: this implies that the functions are subharmonic not only on complex lines but on every holomorphic curve as well, and allows us to define the class also on a complex manifold.

I have understood that the mathematicians around Nicolas Bourbaki were of the opinion that Lelong’s class of functions was a rather uninteresting generalization of the subharmonic functions of one complex variable. But they had to change their opinion when it turned out that this class of functions played an important role in the theory for the  $\bar{\partial}$  operator which Lars Hörmander developed (1965), and which in a natural way came to be used as weight functions in the Hilbert spaces of functions and differential forms that Lars constructed in order to solve a number of classical problems in several complex variables.

## The Lelong number, or *le nombre densité*

The mathematical object which is most associated with Lelong’s name is certainly the Lelong number of a plurisubharmonic function at a point. It is a generalization

of the multiplicity of a zero to a holomorphic function and can be defined in several ways: as a density or as the slope of a convex function.

The Laplacian  $\Delta f$  of a plurisubharmonic function  $f$  is a measure, and this measure has a well-defined mass  $\int_{B(c,r)} \Delta f$  in the open ball  $B(c,r)$  with center at  $c$  and radius  $r$ ; it is an increasing function of  $r$ . In his paper (1950), Lelong showed that even if you divide this mass by the volume of the ball  $B(0,r) \cap \mathbf{C}^{n-1}$  of dimension  $2n - 2$ , the result is an increasing function of  $r$ , i.e., the quotient  $\int_{B(c,r)} \Delta f / \mathbf{vol}(B(0,r) \cap \mathbf{C}^{n-1})$ , which is the mean density of  $\Delta f$  in the ball, is an increasing function of  $r$ . (Density is mass per unit volume, but you have to take the volume in the correct dimension.) Hence the limit

$$\nu_f(c) = \lim_{r \rightarrow 0} \frac{\int_{B(c,r)} \Delta f}{\mathbf{vol}(B(0,r) \cap \mathbf{C}^{n-1})}$$

exists. This number, thus the pointwise density at  $c$ , Lelong called *le nombre densité* ‘the density number’, while everybody else calls it the *Lelong number* of  $f$  at the point  $c$ . Lelong himself most often avoided the term *le nombre de Lelong*, and wrote usually *le nombre densité*; sometimes he said *le nombre vous savez* ‘the number you know’.

In another definition one considers the supremum  $g(t)$  of  $f$  over the ball  $B(c, e^t)$  with radius  $r = e^t$ . It then turns out that  $g$  is an increasing convex function of  $t$ , and the limit  $\lim_{t \rightarrow -\infty} g(t)/t$  therefore exists. This limit is equal to  $\nu_f(c)$ .

The Lelong number has been shown to be a very important entity in complex analysis and has been generalized in several ways, for instance to closed positive currents. An important connection to analytic sets is that the set where the Lelong number of a plurisubharmonic function is at least equal to a certain constant is an analytic set: this is Siu’s theorem, proved by Yum-Tong Siu (1974).

### Integration on an analytic set

An analytic set is locally the common zero set of a family of holomorphic functions: one can describe it as an analytic manifold with singularities. A fundamental problem was to integrate a differential form which is defined on such a set even over the singular points.

Lelong solved this problem (1957a, 1957b): he took integration over the regular points, considered this operation as a current in the sense of Georges de Rham (1903–1990), and then extended it in a suitable manner over the whole analytic set. He introduced closed positive currents and proved results on how to extend them. These currents later became much studied objects in complex analysis; they belong to the supple objects I already mentioned.

### The indicator of an entire function

An entire function grows at infinity; to measure this growth, mathematicians have for a long time used the concept of order and type. But these say nothing about

how the function behaves in different directions: they take account only of the worst direction. For example, we know that the cosine function  $\cos z = \frac{1}{2}(e^{iz} + e^{-iz})$ ,  $z \in \mathbf{C}$ , can be estimated by  $|\cos z| \leq e^{|\operatorname{Im} z|}$ : it grows fast in the imaginary directions but is bounded in the real ones. The logarithm can be estimated by  $\log |\cos z| \leq |\operatorname{Im} z|$ , a convex function of  $z$ . In the estimate  $|\operatorname{Im} z| \leq |z|$  the left-hand side but not the right-hand side is sensitive for the choice of direction.

In several variables one has for instance the entire function  $h(z) = \cos \sqrt{z_1^2 + z_2^2}$ ,  $z = (z_1, z_2) \in \mathbf{C}^2$ , which satisfies

$$\log |h(z)| = \log \left| \cos \sqrt{z_1^2 + z_2^2} \right| \leq \left| \operatorname{Im} \sqrt{z_1^2 + z_2^2} \right| = \varphi(z),$$

where the right-hand side no longer is a convex function: we have for instance  $\varphi(i, \pm 1) = 0$ , while  $\varphi(i, 0) = 1$ . We do have an estimate

$$\left| \operatorname{Im} \sqrt{z_1^2 + z_2^2} \right| \leq \sqrt{|z_1|^2 + |z_2|^2} = \|z\|_2,$$

where the left-hand side but not the right-hand side is sensitive for the direction in which one goes out to infinity.

To describe the growth in different directions, one defines the *indicator*  $p_h$  of an entire function  $h$  of exponential type by

$$p_h(z) = \limsup_{t \rightarrow \infty} \frac{1}{t} \log |h(tz)|, \quad z \in \mathbf{C}^n.$$

It is an important result that the smallest upper semicontinuous majorant  $p_h^*$  of  $p_h$  is plurisubharmonic, where

$$p_h^*(z) = \limsup_{w \rightarrow z} p_h(w), \quad z \in \mathbf{C}^n.$$

Furthermore it is clear that  $p_h^*$  is positively homogeneous:  $p_h^*(tz) = tp_h^*(z)$  for  $t > 0$ ,  $z \in \mathbf{C}^n$ . The question is now whether every positively homogeneous plurisubharmonic function  $f$  is the indicator of some entire function, i.e., whether there exists  $h$  such that  $p_h^* = f$ . Lelong (1966) proved that this is so under an additional condition, viz. that  $f$  is complex homogeneous,  $f(tz) = |t|f(z)$ ,  $t \in \mathbf{C}$ ,  $z \in \mathbf{C}^n$ .

André Martineau (1966, 1967) and I (1967) solved the general problem when  $f$  is only positively homogeneous. Martineau did it even more generally for functions of finite order and finite type. Lelong wrote in his *Notice* (1973:17): “j’ai été devancé simultanément par le suédois C.-O. Kiselman et le regretté A. Martineau” ‘I was overtaken simultaneously by the Swede C. O. Kiselman and by A. Martineau, unfortunately deceased’.

## Séminaire Lelong

For many years, Lelong led a series of seminars known as the *Séminaire Lelong*.

The seminar talks were published. The first seven volumes were from the years 1957/58 through 1966/67 and were published by the Faculty of Sciences in Paris under the title *Séminaire d'Analyse dirigé par Pierre Lelong*. After that they came out in Springer's series *Lecture Notes in Mathematics*, in a total of 15 volumes. The first nine (with the numbers 71, 116, 205, 275, 332, 410, 474, 524, 578) had the title *Séminaire Pierre Lelong (Analyse)*; the next three (694, 822, 919) were called *Séminaire Pierre Lelong – Henri Skoda (Analyse)*, and the final three (1028, 1198, 1295) *Séminaire d'Analyse P. Lelong – P. Dolbeault – H. Skoda*. The total was 22 volumes from the years 1957/58 through 1985–1986. Together these volumes provide us with an impressive survey of the development of several branches of complex analysis during thirty years.

Lelong's seminars always started at 13:45, while all other Paris seminars after lunch started at 13:30. Lelong claimed that one did not have time to have lunch when one had to go to a seminar which started that early. For that very reason, he gave himself—and all other participants—these extra, obviously very important, 15 minutes.

## Meetings with Pierre Lelong

I have met Pierre Lelong many times during forty years, 1966–2005, mostly in France and Sweden, but also in other countries, and will mention here only a few of these occasions.

The first time I met him was in 1966, when we both participated in a four-week conference in La Jolla, *American Mathematical Society 1966 Summer Institute on Entire Functions and Related Parts of Analysis*, June 27 – July 22, 1966. His wife at the time, Jacqueline Lelong-Ferrand (before marriage Jacqueline Ferrand) was also there. They later divorced, and Pierre married France Fages (now France Lelong).<sup>1</sup> Both Jacqueline Ferrand and France Lelong are mathematicians.

The second time was in March 1968, when he invited me to talk at his seminar in Paris. At the time I was a visiting professor at Nice, invited by André Martineau (1930–1972). I gave my lecture on March 13, and talked later three more times at Lelong's seminar.

Pierre Lelong served as Faculty Opponent when Urban Cegrell presented his PhD Thesis in Uppsala on May 23, 1975.

In May 1981, Gérard Cœuré and Henri Skoda organized a big conference in Wimereux in northern France on the occasion of Lelong's retirement. This conference attracted many outstanding scientists; among them Eric Bedford, Henri Cartan, Klas Diederich, Seán Dineen, Pierre Dolbeault, Hans Grauert, Joseph J.

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<sup>1</sup>When I many years later told him that I had divorced and now was with another woman, I was met with a marked approval: it is OK to divorce and remarry once, *mais pas plus que ça* 'but not more than once'.

Kohn, Paul Malliavin, Reinhold Meise, Leopoldo Nachbin, Bernard Shiffman, Nessim Sibony, Józef Siciak, Yum-Tong Siu, Wilhelm Stoll, and Vasiliĭ Sergeevič Vladimirov. According to some participants, the food was not so good (I did not notice). Towards the end of the conference, Lelong, pulled Larry Gruman into a corner and said: “Gruman, on aurait mieux fait d’organiser ce colloque chez vous” ‘Gruman, it would have been better to organize this conference at your place’ (Larry Gruman, personal communication 2011-10-30). This did not refer at all to the organization of the meeting, nor to the mathematics; it was all about the food. Larry lived at the time in the Department of Gers in southern France, the *cuisine* of which Pierre Lelong appreciated during his many visits there.<sup>2</sup>

Lelong received the degree of *Doctor Honoris Causa* at Uppsala University on June 5, 1981. He was appointed to give the acceptance speech for all the seventeen honorary doctors at the banquet in Uppsala Castle. His speech was a brilliant show of French rhetoric. He started off by complaining about the pain he had suffered from puttin’ on the Ritz—a complaint, however, which was so well hidden behind polite words that very few of his hosts or fellow dinner guests understood it as a complaint. Then there came a flood of high-strung praise for science in general and mathematics in particular:

Puis-je dire que pour moi elle est la plus profondément humaine des sciences, la plus universelle aussi et qu’elle est aussi celle où les désirs de l’imagination, et parfois même sa fantaisie, trouvent le mieux à s’accomplir, précisément fortifiées par la présence des règles de la logique? ‘Am I allowed to say that it [mathematics] for me is the most profoundly human of the sciences, and the most universal as well, and that it is also the one where the wishes of our imagination, sometimes even its fantasies, can be best realized, reinforced by the presence of the rules of logic?’ (Pierre Lelong, June 5, 1981)

But he then aired criticism against Sweden’s research policy:

Puis-je dire, Monsieur le Recteur, que cette excellence des recherches et cette place que tiens votre pays dans le domaine des mathématiques ne vous empêchent pas d’être bien sévères : on m’assure que vous maintenez longtemps dans l’attente d’un poste de professeur de jeunes docteurs dont l’étranger a reconnu le mérite [?] ‘Am I allowed to say, Mr President, that this excellence in research and the position that your country occupies in the domain of mathematics do not prevent you from being quite severe: I am being assured that you keep waiting for a long time for a professorship young doctors who have been recognized in other countries as worthy[?]’. (Pierre Lelong, June 5, 1981)

The following day, we went on a boat excursion in the Swedish archipelago, starting

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<sup>2</sup>Gers is situated in the historical region Armagnac, which is part of the historical province and duchy Gascogne. Goose liver (*foie gras*) and armagnac are important products. On Gers *Le petit Larousse* (1967) writes: “Le Gers est un département essentiellement agricole, où la population, dispersée en hameaux et en fermes, pratique une polyculture complexe [...]” ‘Gers is essentially a department for agriculture, where the population, which is spread out in hamlets and farms, practice a complex polyculture [...]’. Lelong’s life testifies to the fact that there is but a small step from *polyculture complexe* to *analyse pluricomplexe*.

at Spillersboda: Urban had a big boat, a renovated fishing boat from the northern coast of the province of Uppland, and took Pierre Lelong, Dan Shea, Bengt Josefson and me on a tour. We visited the National Park of Ångsö, one of the well-preserved islands not far from Uppsala and Stockholm.

As already mentioned, Lelong was a member of the jury for Leif Abrahams-son when Leif presented his PhD Thesis on November 13, 1982. During the week November 7–14, he combined this with a tour of Sweden: a visit to Lund (Lars Gårding and Lars Hörmander), Uppsala (Urban Cegrell, Christer Kiselman), Stockholm (Lars Inge Hedberg), Linköping (Bengt Josefson), and back to Uppsala again for Leif's PhD defense. All was paid for by the Ministry of Foreign Affairs of France.

In 1986, Lelong paid a visit to Sweden within the framework of an exchange program between the French and Swedish Academies of Science. He visited Umeå University, Uppsala University and the Academy of Sciences in Stockholm during the week October 2–9, 1986.

Lawrence Gruman (better known as Larry) worked with Lelong, and they wrote a book (Lelong & Gruman 1986) on entire functions. It contains a lot about the growth at infinity of these functions, functions of regular growth, solutions to the equation  $i\partial\bar{\partial}U = \theta$ , relations between the growth of a function and the growth of the area of its zero set, etc.

The person who last worked with Lelong is Alexander Rashkovskii, now in Stavanger. They published a joint paper in 1999, Alexander translated a text by Lelong into English, and they discussed a lot.

On the occasion of Larry's retirement, Paul Sabatier University in Toulouse organized a conference in May 2002, where Lelong appeared in high spirits and gave a mathematical lecture.

At a conference in Paris for Henri Skoda in September 2005, Lelong gave a long speech for Henri at a reception on September 15, but no mathematical lecture. The first day of the conference, September 12, he was present during the morning session. I talked with him during the lunch, and accompanied him to his car, an ostensibly damaged Citroën BX. He drove home alone in the Paris traffic, 93 years old.

## Charles de Gaulle

General Charles de Gaulle (1890–1970) was installed as President of the Fifth Republic on January 8, 1959. The same day he appointed Pierre Lelong as *Conseiller technique au Secrétariat général de la Présidence de la République* 'Technical Counselor at the General Secretariat of the President of the Republic'. Lelong's responsibility was *Recherche scientifique, Éducation nationale et Santé publique* 'Scientific Research, Public Education [at all levels, not only higher education], and Public Health', thus encompassing several ministries. Lelong had this function during two years, up to January 8, 1961. He could then obtain that a biologist, R. Camus, a professor at the University of Paris, took over the post as counselor. (It seems it was not at all obvious that he could get a successor at that post.) When Camus

left in 1964, the post was suppressed, which meant that the sciences no longer had direct access to the General. (Lelong 1977:212.)

Lelong gives an insight into the working habits of the General, who left a lot of initiative to his Technical Counselor during the two years of the latter's tenure. From time to time, a note from the Counselor sufficed to start a government action—provided (of course!) that the note was short and well written. When the note was approved and annotated in de Gaulle's handwriting, it went to the General Secretariat and from there to the relevant ministry with the understanding that a "proposal" from the ministry along these lines would be welcomed by the president. A strange procedure, writes Lelong, no doubt conditioned by the technical context, but even more so by the General's personality. (Lelong 1977:209–210.) An example is the *Direction des recherches et des moyens d'essai* (D.R.M.E.), created at the end of 1960 and approved by the General following a report from the Scientific Counselor, viz. Lelong, (1977:198).

The last years of the Fourth Republic, before de Gaulle and the Fifth Republic, were characterized by weak, short-lived governments. A lot of public affairs got stuck for years for lack of forceful national leadership.

When de Gaulle came to power many matters started to move and a series of important decisions could be made. Lelong found himself in the middle of these, actually already during the Fall of 1958 before de Gaulle had become president and he himself had been formally appointed: The Constitution of the Fifth Republic came into effect on October 4, 1958, following a referendum on September 28. An ordinance of November 28, 1958, established a new structure for the government's handling of research (Lelong 1977:186). In a statement which I find typical, Lelong mentions that it is easy to create a research institution but difficult to put an end to it (1977:197).

In addition to the *Comité consultatif* mentioned below, de Gaulle created the *Délégation à la recherche scientifique et technique* (D.R.S.T.) attached to the office of the Prime Minister (Lelong 1979:180).

In his contribution to a book on de Gaulle's collaborators, he describes how de Gaulle paid a lot of attention to high-tech industries in four domains: nuclear energy; civil aviation; space research; and computers and IT (Lelong 1979:179).

### **La Halle aux vins**

One of the affairs that had got stuck during the Fourth Republic was the problem around *La Halle aux vins*, a block devoted to the selling of wine since 1665, most centrally located in the Fifth Arrondissement in Paris. The Paris University wished to come to this district, but the wine merchants had blocked this aspiration for many years.

With de Gaulle there came a solution, and the university could take over almost the whole block. For a long time it was called "la Faculté des Sciences de la Halle aux vins." Nowadays Paris VI and Paris VII are there, as well as the *Institut du monde arabe*. I believe that Lelong played an important role for the decision to liberate the block for education and research.



## Advice to the president

One evening Lelong was correcting students' exams at his home when he received a phone call from the Élysée Palace. He drove there. The General explained the problem, which was about the Catholic schools: should they receive financial support from the state?

For a long time, the French Republic has been secular: The state shall not be involved in or support any religion. The *laïcité* is inscribed in the French constitution of 1958 as well as in a law of 1905.

Lelong thought about the problem—it could not have been for a long time, for de Gaulle wanted an immediate answer. Lelong replied that he was of the opinion that the private Catholic schools should receive support from the state. The General: “Je suis de votre avis.” ‘I share your opinion.’ The meeting was over. The question was resolved. Lelong drove home and continued to correct the students' papers.

## Planning of research

In a long article, Lelong (1964) describes his views on the evolution of science and the planning of research. It is remarkable for its sheer length (62 pages) and the fact that it appeared in a journal devoted to economics, but above all for the broad and at the same time very detailed picture the author presents on the possibilities for a state (France) to initiate, coordinate, plan, and steer scientific and technical research, as well as the obstacles for this activity.

Pierre Lelong was during the four years December 1960 through December 1964 a member of the *Comité consultatif de la Recherche scientifique et technique*, which had twelve members, and he was its chairman during two years, December 1961 through December 1963 (Lelong *Notice* 1973:4; cf. « *L'Entourage* » et de *Gaule* 1979:370). The article (1964), written when he still chaired the committee, is clearly an attempt to influence the authors behind the upcoming Fifth Plan based on the experience he had had concerning the Fourth Plan (1961–1965) and its shortcomings (e.g., 1964:48). In the paper several pages (1964:10–16) are devoted to a description of the financing of research in the United States, which is presented as a model for French research policy.

The article is obviously not written for scientists; it is aimed at politicians. He mentions even twice the basic conditions for research, well known to scientists but perhaps not to politicians: *le mécanisme de l'invention est complexe, peu exploré et, ne l'oublions pas, il n'existe pas de recherche véritable sans chercheurs ni aléas inhérents à la recherche* ‘the mechanism of invention is complex, little explored, and, let us not forget it, there is no real research without researchers and the hazards inherent in research’ (1964:2); and again: *On ne fait pas de recherche sans chercheur qualifié* ‘One cannot do research without a qualified researcher’ (1964:27). He admits that it is probably not possible to plan scientific activity (1964:4). He describes eloquently the obstacles to an orderly development, *Nombre de freins* ‘A number of brakes’, that are of several different kinds (1964:37–40).

This long article, which curiously is not mentioned in the bibliography in Lelong's *Notice* (1973), is indeed remarkable for its political and philosophical contents. It also provides an explanation of the fact that its author could reach positions of great political importance, e.g., as a technical counselor to the first President of the Fifth Republic.

## Georges Pompidou

Also after he had left the function as Counselor to de Gaulle, Lelong had many commissions from the highest offices of the French Republic. Georges Pompidou (1911–1974) was Prime Minister during the years 1962–1968, and President of the Republic during 1969–1974. After the period of counselor to de Gaulle, Lelong was charged with preparing the budget for all research in France for Pompidou when the latter was Prime Minister (1993:2). He writes:

Je puis dire aujourd'hui que le projet spatial fut par mes soins retardé d'un an pour faire passer le projet de la biologie moléculaire. 'I can say today that space research was delayed by one year because of my actions, to allow molecular biology to advance.' (Lelong 1993:2)

Lelong and Pompidou said *tu* to each other. Pompidou was less than one year older than Lelong and entered the *École normale supérieure* in 1931, the same year as Lelong. They were school mates and therefore used the informal way of addressing each other for the rest of their lives.<sup>3</sup> To make this clear to me, Lelong, when he recounted something Pompidou had said, never used indirect quotes (And then Pompidou said to me that I should do this or that) but always direct quotes: And then Pompidou said to me: "Tu dois faire ceci et cela."

Also in writing, Lelong used direct quotes to make the intimacy of their relation evident. He quotes Pompidou like this:

«Penses-tu qu'il faut vraiment faire l'Espace ?» me disait-il alors qu'il était Premier Ministre. "Do you think that we really have to do Space?" [That is, to accept a large space research program] he said to me when he was Prime Minister.' (Lelong 1993:2)

So the resistance against space research was perhaps there with Pompidou already before Lelong presented the budget.

## Words of wisdom

Without having been asked, Pierre Lelong gave me and others advice on how to view one's retirement: one should pretend that nothing has happened and go on doing research just as usual. And actually, more than a quarter of Lelong's mathematical papers were published after he had reached the age of 69.

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<sup>3</sup>It is perhaps unnecessary to add that something similar cannot be imagined concerning de Gaulle, who was reported to say *vous* to his own wife, Yvonne.

The other word of wisdom that comes to my mind is this: The highest honor a mathematician can get in this world is that his or her results become so well known and generally accepted that everybody thinks that they are completely self-evident and trivial. And this is probably so: you cannot understand mathematics, you can only get used to it, and what you are used to, you consider as self-evident. Your brain has been re-programmed.

In writing he comes close to this idea in a speech in Wimereux in May 1981:

En mathématiques l'indispensable processus d'assimilation est un processus de trivialisat[i]on. 'In mathematics the unavoidable process of assimilation is one of trivialization.' (Lelong 1982:191)

The plurisubharmonic functions are perhaps just such a self-evident structure.

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