2-representations of finitary 2-categories

(joint work with Vanessa Miemietz)

Volodymyr Mazorchuk
(Uppsala University)

Category Theoretic Methods in Representation Theory
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2-categories

Note:
All categories in this talk are assumed to be locally small (or small if necessary).

Definition:
A 2-category is a category enriched over the monoidal category $\text{Cat}$ of small categories.
That is:
- A 2-category consists of:
  ▶ a class (or set) $C$ of objects;
  ▶ for every $i, j \in C$ a small category $C(i, j)$ of morphisms from $i$ to $j$ (objects in $C(i, j)$ are called 1-morphisms of $C$ and morphisms in $C(i, j)$ are called 2-morphisms of $C$);
  ▶ functorial composition $C(j, k) \times C(i, j) \to C(i, k)$;
  ▶ identity 1-morphisms $1_i$ for every $i \in C$;
  ▶ natural (strict) axioms;

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- The category $\mathbf{Cat}$ of small categories (1-morphisms are functors and 2-morphisms are natural transformations);
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- the full subcategory $A_k^f$ of $A_k$ consisting of small fully additive $k$-linear categories with finitely many indecomposable objects up to isomorphism;

- the category $R_k$ of small categories equivalent to module categories of finite-dimensional associative $k$-algebras;
2-category $\mathcal{P}$ of Soergel bimodules
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\[ \mathcal{C} = \mathcal{C}_n = \mathbb{C}[x_1, \ldots, x_n]/(I_n) \] – the coinvariant algebra of $S_n$

$I_n$ – the set of homogeneous $(S_n)$ symmetric polynomials of positive degree;
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define by induction on $k$ the **Soergel $\mathcal{C}$-bimodule** $B_w$ as follows:

$B_e = \mathcal{C}$ and $B_w$ as the unique direct summand of $\hat{B}_w$ not yet defined;
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$$A = A_1 \oplus A_2 \oplus \cdots \oplus A_k$$

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2-morphisms are natural transformations
2-representations

Definition. A 2-representation of a 2-category $C$ is a 2-functor (i.e. a functor respecting the 2-structure) to some "classical" 2-category. 2-representations of $C$ (into a fixed category) together with 2-natural transformations and modifications form a 2-category. For a $k$-linear 2-category $C$ we have:

- additive representations $C\rightarrow \text{mod}_{A}$ into $A$
- finitary representations $C\rightarrow \text{afmod}_{A}$ into $A_{f}$
- abelian representations $C\rightarrow \text{mod}_{R}$ into $R$

Example. The 2-category $CA$ was defined via its defining representation.
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**Example.** The 2-category $\mathcal{C}_A$ was defined via its defining representation.
Fiat categories

Definition. A 2-category $C$ is called fiat (initiary - involution - adjunction - two category) provided that the following conditions are satisfied:

- $C$ has finitely many objects;
- each $C(i, j) \in A_k$;
- composition is biadditive and $k$-linear;
- all $k$-spaces of 2-morphisms are finite dimensional;
- all $1_i$ are indecomposable;
- $C$ has a weak involution $^*$;
- $C$ has adjunction morphisms $F \circ F^* \to 1_i$ and $1_j \to F^* \circ F$.

Examples. $S$ is fiat; $C_A$ is fiat if and only if $A$ is self-injective and weakly symmetric (i.e. the top and the socle of each indecomposable projective are isomorphic).
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Principal 2-representations
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**Definition.** For $i \in \mathcal{C}$ the corresponding **principal** 2-representation $\mathbb{P}_i$ of $\mathcal{C}$ is defined as the 2-functor

$$\mathcal{C}(i, -) : \mathcal{C} \to \mathcal{A}_f^f.$$
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**Yoneda lemma.** For any $M \in \mathcal{C}$-amod we have

$$\text{Hom}_{\mathcal{C}}(\mathbb{P}_i, M) = M(i).$$
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Abelianization

**Definition.** The **abelianization** 2-functor \( \tilde{\cdot} : \mathcal{C}\text{-afmod} \to \mathcal{C}\text{-amod} \) is defined as follows:
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given \( M \in \mathcal{C}_{\text{afmod}} \) and \( i \in \mathcal{C} \) the category \( \tilde{M}(i) \) has objects
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and morphisms

$$X \xrightarrow{\alpha} Y \quad \text{modulo} \quad X' \xrightarrow{\alpha'} Y'.$$
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Given \( M \in \mathcal{C} \text{-afmod} \) and \( i \in \mathcal{C} \), the category \( \overline{M}(i) \) has objects \( X, Y \in M(i), \; \alpha : X \to Y \); and morphisms

\[
\begin{array}{ccc}
X & \xrightarrow{\alpha} & Y \\
\downarrow{\beta} & & \downarrow{\gamma} \\
X' & \xrightarrow{\alpha'} & Y'
\end{array}
\]

and morphisms modulo

\[
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The 2-action of \( \mathcal{C} \) is defined componentwise.
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extends to a 2-functor componentwise
Multisemigroups

**Definition.** A **multisemigroup** is a pair \((S, \diamond)\), where \(S\) is a set and \(\diamond : S \times S \rightarrow 2^S\) is associative in the sense

\[
\bigcup_{s \in a \diamond b} s \diamond c = \bigcup_{t \in b \diamond c} a \diamond t, \quad \text{for all } a, b, c \in S
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**Example 1.** Any semigroup is a multisemigroup.
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Definition. A multisemigroup is a pair \((S, \diamond)\), where \(S\) is a set and \(\diamond : S \times S \rightarrow 2^S\) is associative in the sense

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\bigcup_{s \in a \diamond b} s \diamond c = \bigcup_{t \in b \diamond c} a \diamond t, \quad \text{for all } a, b, c \in S
\]

Example 1. Any semigroup is a multisemigroup.

Example 2. \((\mathbb{Z}_+, \diamond)\), where \(\mathbb{Z}_+ = \{0, 1, 2, \ldots\}\) and

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m \diamond n = \{i : |m - n| \leq i \leq m + n; \quad i \equiv m + n \mod 2\}.
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Multisemigroups

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**Green’s relations (Kazhdan-Lusztig cells):**

- \(a \sim_L b\) iff \(S \diamond a = S \diamond b\);
Multisemigroups

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- \(a \sim_J b\) iff \(S \diamond a \diamond S = S \diamond b \diamond S\)
Fiat categories, principal 2-representations and abelianization

Multisemigroup of a fiat category

$F, G$ are composable indecomposable 1-morphisms in $\mathcal{C}$, then

$$F \circ G \cong \sum_{H \text{ indec.}} m^H_{F,G} H.$$
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**Definition.** The multisemigroup \((S(\mathcal{C}), \diamond)\) of a fiat category \( \mathcal{C} \) is defined as follows: \( S(\mathcal{C}) \) is the set of isomorphism classes of 1-morphisms in \( \mathcal{C} \) (including 0),

\[
[F] \diamond [G] = \begin{cases} 
[H]: m_{F,G}^{H} \neq 0, & F \circ G \text{ defined and } \neq 0; \\
0, & \text{else}.
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\]
Multisemigroup of a fiat category

$F, G$ are composable indecomposable 1-morphisms in $C$, then

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0, & \text{else.}
\end{cases}$$

Sometimes $S(C)' := S(C) \setminus \{0\}$ is closed with respect to $\diamond$. 

Volodymyr Mazorchuk (Uppsala University)
Multisemigroup of a fiat category

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**Example.** \( \mathcal{C}_{\mathfrak{sl}_2} \) – the 2-category of the tensor category of finite dimensional \( \mathfrak{sl}_2 \)-modules.
Multisemigroup of a fiat category

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\( S(\mathcal{C}_{\mathfrak{sl}_2})' \xleftrightarrow{1:1} \mathbb{Z}_+ \) (via highest weight) and \((S(\mathcal{C}_{\mathfrak{sl}_2})', \diamond) \cong (\mathbb{Z}_+, \diamond)\)
Further examples

Soergel bimodules.
Further examples

**Soergel bimodules.**

$S(\mathcal{I})' \leftrightarrow S_n$
Further examples

Soergel bimodules.

\[ S(\mathcal{P})' \leftrightarrow S_n \]

under this identification left cells of \( S(\mathcal{P})' \) correspond to right cells of \( S_n \) and vice versa
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The fiat category $\mathcal{C}_A$, $A = A_1 \oplus \cdots \oplus A_k$. 
Further examples

Soergel bimodules.

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**The fiat category** \( \mathcal{C}_A, A = A_1 \oplus \cdots \oplus A_k \).

two-sided cells: \( \{ 1_1 \}, \{ 1_2 \}, \ldots, \{ 1_k \}, J := \{ A_i e \otimes_{k} fA_j : e, f \text{-primitive} \} \)
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Volodymyr Mazorchuk (Uppsala University)
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Further examples

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two-sided cells: $\{1_1\}, \{1_2\}, \ldots, \{1_k\}$, $J := \{A_i e \otimes_k f A_j : e, f$-primitive\}

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note: $A_j f \otimes_k e A_i \otimes_A A_i e \otimes_k f A_j \cong \text{dim}(A_i e) A_j f \otimes_k f A_j$ and $\text{dim}(A_i e)$ is constant on a right cell!!!
Duflo involution of a left cell

\( \mathcal{C} \) – fiat category; \( \mathcal{L} \) – left cell of \( \mathcal{C} \)
Duflo involution of a left cell

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there is \( i \in \mathcal{C} \) such that every \( F \in \mathcal{L} \) belongs to some \( \mathcal{C}(i, j) \)
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consider \( \overline{P}_i \) and for an indecomposable 1-morphism \( F \in \mathcal{L} \cap \mathcal{C}(i, j) \)

denote by \( P_F \) the projective object \( 0 \to F \) of \( \overline{P}_i(j) \) and by \( L_F \) the simple top of \( P_F \)
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**Proposition.**

1. There is a unique \( K \subset P_{\perp i} \) such that \( F P_{\perp i} / K = 0 \) for any \( F \in \mathcal{L} \)
   while \( F X \neq 0 \) for any \( X \in \text{top}(K) \).
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Proposition.

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2. \( K \) has simple top \( L_{G_L} \).
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2. \( K \) has simple top \( L_{G_{\mathcal{L}}} \).
3. Both \( G_{\mathcal{L}} \) and \( G_{\mathcal{L}}^* \) belong to \( \mathcal{L} \).
Duflo involution of a left cell

$\mathcal{C}$ – fiat category; $\mathcal{L}$ – left cell of $\mathcal{C}$

there is $i \in \mathcal{C}$ such that every $F \in \mathcal{L}$ belongs to some $\mathcal{C}(i, j)$

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**Proposition.**

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**Definition.** $G_L$ is the Duflo involution in $\mathcal{L}$
Definition of a cell 2-representation

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Definition of a cell 2-representation

\( \mathcal{C} \) – fiat category; \( \mathcal{L} \) – left cell of \( \mathcal{C} \); \( G_{\mathcal{L}} \) – Duflo involution

**Theorem.** \( \mathcal{X} := \text{add}\{F L_{G_{\mathcal{L}}}: F \in \mathcal{L}\} \) is closed under the action of \( \mathcal{C} \)
Definition of a cell 2-representation

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**Definition.** The **cell 2-representation** of \( \mathcal{C} \) corresponding to \( \mathcal{L} \) is the finitary 2-representation obtained by restricting the action of \( \mathcal{C} \) to \( \mathcal{X} \).
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**Definition.** Two 2-representations of \( \mathcal{C} \) are called **elementary equivalent** if there is a homomorphism between them which is an equivalence when restricted to every \( i \in \mathcal{C} \).

**Definition.** Two 2-representations of \( \mathcal{C} \) are called **equivalent** if there is a finite sequence of 2-representations starting with the first one and ending with the second one such that every pair of neighbors in the sequence are elementary equivalent.
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Comparison of cell 2-representation

Main theorem.
Comparison of cell 2-representation

Main theorem.

Let $\mathcal{J}$ be a 2-sided cell of $\mathcal{C}$ such that:

- different left cells inside $\mathcal{J}$ are not comparable w.r.t. the left order;
- for any $L, R \subseteq \mathcal{J}$ we have $|L \cap R| = 1$;
- the function $F \mapsto \text{m} F \circ \text{m} F$, where $F \ast \circ F = \text{m} F \circ \text{H}$ is constant on right cells of $\mathcal{J}$.

The for any two left cells $L$ and $L'$ of $\mathcal{J}$ the corresponding cell 2-representations are equivalent.

Example. Works for both $\mathcal{C}$ (in type $\mathcal{A}$) and $\mathcal{C}_A$. 

Volodymyr Mazorchuk (Uppsala University)
Comparison of cell 2-representation

**Main theorem.**

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Comparison of cell 2-representation

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Example. Works for both $C$ (in type A) and $C_A$. 

Volodymyr Mazorchuk (Uppsala University)