A small variation of Aczel's Relation Reflection Scheme

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Relation Reflection Scheme (Aczel) Let X and $R \subseteq X \times X$ be classes. Let $a \subseteq X$ be a subset. If $R : X \succ X$, then there is a subset $a \subseteq b \subseteq X$ so that $R : b \succ b$.

This scheme may be reformulated as a scheme resembling Dependent Choice.

Multivalued Dependent Choice: Let X and $R \subseteq X \times X$ be classes. Let $a \subseteq X$ be a subset. If $R : X \succ X$, then there is a function $f : \mathbb{N} \to \mathcal{P}(X)$ so that $f(0) \supseteq a$ and $R : f(n) \succ f(n+1)$ for all n.

Here $\mathcal{P}(X)$ is the class of subsets of X.

Theorem: RRS $\Longleftrightarrow$ MDC.

Proof: Let X and $R \subseteq X \times X$ be classes. Let $a \subseteq X$ be a subset. Suppose that $R : X \succ X$.

($\Rightarrow$) If RRS holds then there is a set $a \subseteq b \subseteq X$ such that $R : b \succ b$. Then take $f(n) = b$ for all n.

($\Leftarrow$) Suppose MDC holds. Then use the function f obtained from this scheme and let $b = \cup_n f(n)$. This is a set and clearly $R : b \succ b$. $\Box$