Identity Types - Topological and Categorical Structure

Workshop, Uppsala, November 13-14, 2006

Venue: MIC and Ångström laboratory, Polacksbacken, Uppsala

Programme

Monday 13 November

Morning session (Room 6140, MIC)

9.30 - 10.15	M. Hyland	Algebraic homotopy theory: lessons for type theory

Coffee

10.45 -11.30	T. Streicher	Identity types vs. weak omega-groupoids: some ideas, some problems
11.40 -12.05	W. Chacholski	Representations of spaces and mapping complexes

Lunch break

Afternoon session (Room Å4004, Ångström lab.)

14.00 -14.45	S. Awodey	Type theory of higher-dimensional categories
15.00 -15.45	M.A. Warren	Model categories and intensional identity types

Coffee break

16.15 -17.00	R. Garner	Factorisation systems for type theory
17.15 -18.00	P. Dybjer	History and meaning of the identity type

18.30 *Conference dinner at Eklundshof*

The workshop is organised by Erik Palmgren, with support from the Swedish Research Council (VR) and the mathematics departments of Uppsala University and Stockholm University.

Webpage of the workshop: `www.math.uu.se/~palmgren/itt`

Tuesday 14 November

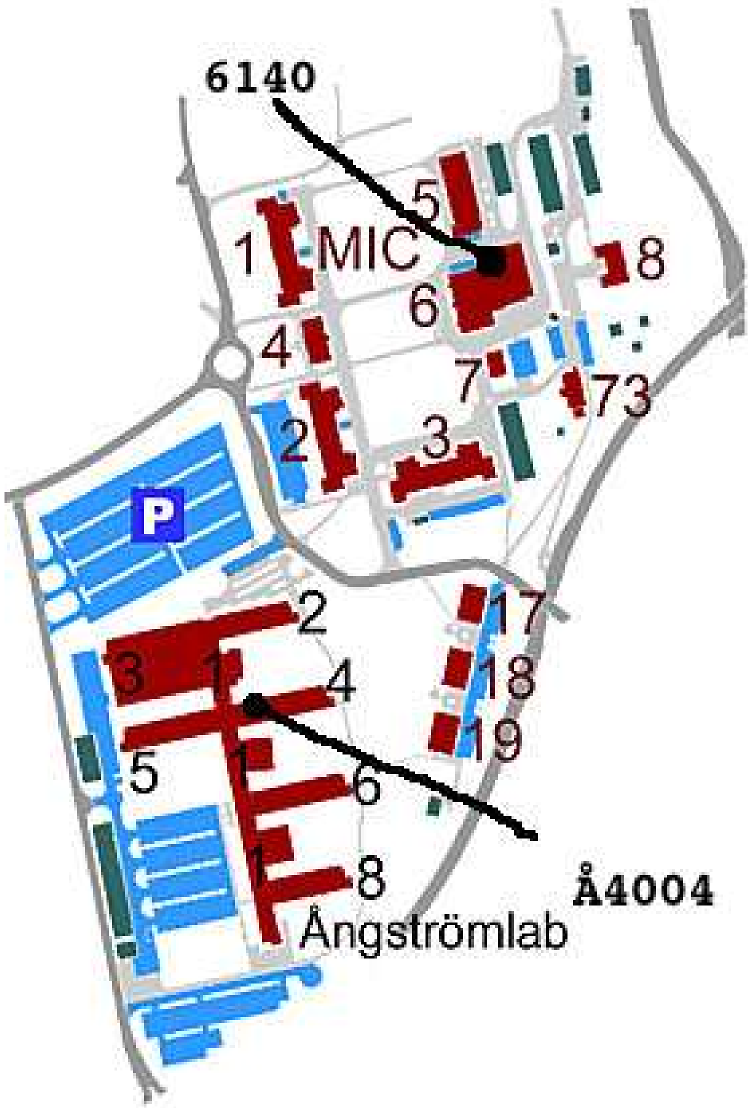
Final session (Room 6140, MIC)

9.00 - 9.45	N. Gambino	Quillen model structures on diagrams of categories
9.55 - 10.40	B. van den Berg	Types as weak omega-categories.

Coffee

10.55 -11.20	M.E. Maietti & G. Sambin	Intensionality vs. extensionality: a solution with two levels of abstraction.
11.30 -12.30	P. Martin-Löf	Topos theory and type theory

Map of polacksbacken, Uppsala



Abstracts of talks

Steve Awodey (Pittsburgh), *Type theory of higher-dimensional categories*

We investigate some structures in higher-dimensional categories that are useful for the semantics of intensional type theory. The possibility of using such type theories as an internal language for certain kinds of higher categories is considered.

Benno van den Berg (Darmstadt), *Types as weak omega-categories*

Fix a type X in a context Γ. Define a globular set as follows: A_0 consists of the terms of type X in context Γ, modulo definitional equality; A_1 consists of terms of the types $Id(X,p,q)$ (in context Γ) for elements p, q in A_0, modulo definitional equality; A_2 consists of terms of well-formed types $Id(Id(X,p,q),r,s)$ (in context Γ) for elements p, q in A_0, r, s in A_1, modulo definitional equality; etcetera...

What is the structure of this globular set? That of a weak omega-groupoid, certainly. But unless I missed something there is no definition of a weak omega-groupoid for globular sets. However, there is a definition of a weak omega-category, especially the one explained in detail in Tom Leinster's book "Higher operads, higher categories". I will show that the globular set defined above is a weak omega-category in precisely this sense.

Wojciech Chacholski (Stockholm), *Representations of spaces and mapping complexes.*

Derived functors of colimits and limits, mapping complexes, fibrant and cofibrant replacements are fundamental objects of homotopy theory. The frameworks which have been developed in order to study these constructions range from model categories to theory of derivators. These approaches however are not easy to use. For example it is often a difficult task to impose a model structure that reflects desired properties. Even if one is able to prove its existence, such a statement shades little info on how to construct cofibrant/ fibrant replacements. In the talk I will explain a modified approach, the key feature of which is its simplicity and applicability. This is joint work with J. Scherer.

Peter Dybjer (Göteborg), *History and meaning of the identity type.*

Contents: Lawvere's equality in hyperdoctrines. Howard's treatment of identity. Martin-Löf's identity in his theory of iterated inductive definitions. The different identity types in versions of type theory. The meaning explanations for the identity type. Identity type vs identity judgement. My own experience of proving coherence of monoidal categories using type theory as a metalanguage. My own view of identity in type theory justifying extensional identity types.

Nicola Gambino (Montreal), *Quillen model structures on diagrams of categories*

The category of small categories admits a natural Quillen model structure whose weak equivalences are categorical equivalences. The aim of this talk is to show how this model structure determines two distinct Quillen model structures on diagrams of categories. I will discuss how this result links with 2-dimensional category theory, the theory of homotopy limits, and Lawvere's notion of hyperdoctrine.

Richard Garner (Uppsala), *Factorisation axioms for type theory*

Though the type constructors of intensional type theory hint at the presence of a weak factorisation system on its category of contexts, they are not sufficiently strong to construct such. This can be fixed by decree, by adding axioms for a weak factorisation system to our type theory. We examine the consequences of doing this.

Martin Hyland (Cambridge), *Algebraic Homotopy Theory: Lessons for Type Theory*

Constructions in abstract homotopy theory generally have two ingredients. One is essentially algebraic and is founded on ideas of enriched category theory. The other is strictly homotopy theoretic in flavour and typically uses ideas from the theory of model categories. I shall explore the impact for type theory of an approach based on the algebraic ideas.

Maria Emilia Maietti & Giovanni Sambin (Padova), *Intensionality vs. extensionality: a solution with two levels of abstraction.*

Some precise results show that extensionality of a theory is incompatible with its effectiveness, when this is meant to include consistency with the axiom of choice and internal Church thesis. This looks as an obstacle to the project of formalization of mathematics in a computer language, since the latter is by definition effective while the former is extensional by tradition. There is a way out (which we first proposed at TYPES '04) which is very natural, although contrary to established tradition. That is to introduce two theories: a ground type theory, which is a sort of idealized programming language and thus is intensional, and an extensional theory, suitable to develop (constructive) mathematics. The novelty lies on the link between the two theories: the extensional theory is to be obtained from the intensional one by abstraction, that is by "forgetting" some information, and this is to be done in such a way that implementation is always possible, that is, by forgetting only those pieces of information which can be restored at will. This requirement is not as trivial as it may look at first: the second incompleteness theorem by Gödel says that the normalization process of a formal system will always be a source to restore information, which is not available within the system itself.

Per Martin-Löf (Stockholm), *Topos theory and type theory*

My purpose is to relate type theory to topos theory by taking the concept of model of type theory to be the counterpart of the notion of elementary topos. Inductive and projective limits of comma (or slice) models of type theory will be considered. The former correspond to the filter quotients of topos theory.

Thomas Streicher (Darmstadt), *Identity Types vs. Weak* ω*-groupoids some ideas, some problems*

After reviewing some basics about Identity Types in intensional Martin-Löf type theory and shortly presenting the groupoid model I explain why every type forms an internal weak ω-groupoid. It thus seems tempting to use Kan complexes as a model. But there are problems with the Beck condition. I sketch an idea how to define a sufficiently internal notion of Kan complexes allowing to choose "fillers" in a functorial way. This notion might be of interest for geometers as well. Finally, I discuss to which extent a weak ω-groupoid model says something about the types definable in traditional type theory. I am afraid nothing because their interpretations all stay within discrete simplicial sets.

Michael Warren (Pittsburgh), *Model categories and intensional identity types*

Quillen model categories provide a flexible axiomatic framework in which to develop the homotopy theory of various categories. In this talk we will show that model categories also provide a framework in which to develop the semantics of Martin-Löf's intensional type theory. In particular, the internal language of any model category, in which fibrant objects are closed types and fibrations are dependent types, contains a form of intensional identity type arising from the path objects present in the model structure. We explore this connection further by considering several variations on this theme as well as looking at dependent products and sums in the setting of Quillen model categories. If time permits the connection between cocategory objects and model structures will also be discussed.