Quotient topologies in constructive set theory and type theory

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Abstract

The standard construction of quotient spaces in topology uses full separation and power sets. We show how to make this construction using only the (generalised) predicative methods available in constructive type theory and constructive set theory.

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1 Introduction

We consider the problem of constructing a neighborhood base for a quotient space $X/\sim$ given a base for the underlying space X. There is a trivial solution to this problem. Take the topology on the quotient space to be the base. The standard construction is to declare that

$$U \subset X/\sim \text{ is open} \iff q^{-1}(U) \text{ is open in } X. \tag{1}$$

Here $q : X \to X/\sim$ is the quotient map, taking an element to the equivalence class it represents.

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This solution is unsatisfactory from the predicative point of view, since the base is defined in terms of all subsets of a given set. In particular, this simple solution is not a priori possible to formalize in constructive set theory or constructive type theory. Below we present two different constructions which are formalizable in those respective theories. The first one (Section 2) uses the type-theoretic choice principle (every set has a projective cover) often used in Bishop's work [2, 3, 6], and a restricted power set operation which is valid in Martin-Löf type theory [5]. The second one (Section 3) employs, apart from standard axioms of CZF, only relativized dependent choice [1]. Lastly, Section 4 shows that the existence of final, or coinduced, topologies follows from the construction of quotient spaces and coproduct spaces, much in the same way as in the classical setting.

2 Construction of bases in type theory

We assume that each set A has a *projective cover*, that is a projective set $\underline{A}$ and surjective function $\iota_A : \underline{A} \to A$. Recall [6] that one, of several equivalent, definitions of a *projective set* P is that the axiom of choice is valid on the set. This means that P has the property that if for any $x \in P$ there is some $y \in B$ satisfying the relation $R(x, y)$, then there is a function $f : P \to B$ so that for all $x \in P$, $R(x, f(x))$.

Next we introduce the notion of *strongly regular family of subsets*. Consider a family of $\mathcal{F} = \{F_s\}_{s\in S}$ of subsets of X, indexed by a set S. For a subset $A \subset X$, write $A \in \mathcal{F}$ if there is some $s \in S$ so that $A = F_s$. The family is I*-closed* if for any function $g : I \to S$, the union $\cup_{a\in I} F_{g(a)} \in \mathcal{F}$. The family $\mathcal{F}$ is *strongly regular* if it is $\underline{A}$-closed for each $A \in \mathcal{F}$.

> **Regularization Axiom (RA).** For any family $\mathcal{B}$ of subsets of X, and any family of sets $\{I_k\}_{k\in K}$ there exists a strongly regular family $\mathcal{F}$ of subsets of X, that includes $\mathcal{B}$, and which is I_k-closed for each $k \in K$.

RA can be justified using the notion of regular universe operator in type theory, see [7].

Remark 2.1. It is clear that the full power set axiom implies RA. Conversely, assuming the principle of excluded middle, and then applying RA to a family

$\mathcal{B}$ including $\emptyset$, X and all singletons $\{x\}$ where $x \in X$, we get that $\mathcal{F} = Pow(X)$.

Theorem 2.2. *Let X be a topological space with a set-indexed neighborhood base. Then each quotient space $X/\sim$ by an equivalence relation $\sim$ has a set-indexed neighborhood base as well.*

Proof. Let $\{U_i\}_{i \in I}$ be a set-indexed neighborhood base for X. For an equivalence relation $\sim$ on X, let $q : X \to X/\sim$ be its quotient map. For a subset A of X, let

$$\widetilde{A} = q^{-1}(q[A]). \tag{2}$$

For any projective cover $\iota : \underline{A} \to A$ note that

$$\widetilde{A} = \bigcup_{a \in \underline{A}} \widetilde{\{\iota(a)\}}. \tag{3}$$

Suppose now, by the Strong Regularity Axiom, that $\mathcal{F} = \{F_s\}_{s \in S}$ is an $\mathbb{N}$-closed, strongly regular family of subsets of X that includes

$$U_i, \quad \{x\}, \quad \widetilde{\{x\}} \tag{4}$$

for every $i \in I$ and $x \in X$. Note that by (3) and the axiom we have

$$A \in \mathcal{F} \Longrightarrow \widetilde{A} \in \mathcal{F}. \tag{5}$$

Let T be the set of $s \in S$ such that F_s is open in X and $\widetilde{F_s} = F_s$. We claim that

$$\{q[F_s]\}_{s \in T}$$

is a base for $X/\sim$.

Since $q^{-1}(q[F_s]) = \widetilde{F_s} = F_s$ and F_s is open, for any $s \in T$, it is clear that the base sets are open in the quotient topology. Conversely, assume that $q^{-1}(V)$ is open in X. Let $q(x) \in V$, i.e. $x \in q^{-1}(V)$. Put $Z_0 = \{x\}$. Then $Z_0 \in \mathcal{F}$ and $Z_0 \subset q^{-1}(V)$.

Suppose that subsets of $q^{-1}(V)$

$$Z_0 \subset Z_1 \subset \cdots \subset Z_n \tag{6}$$

have been constructed in $\mathcal{F}$ such that $Z_1, \ldots, Z_n$ are open and

$$\widetilde{Z_k} \subset Z_{k+1} \qquad (k = 0, \ldots, n-1).$$

Then by (5) $\widetilde{Z_n} \in \mathcal{F}$ and clearly $\widetilde{Z_n} \subset q^{-1}(V)$. Let $A = \widetilde{Z_n}$ and let $\iota : \underline{A} \to A$ be its projective cover. For each $a \in \underline{A}$ there is $s \in S$ such that F_s is a basic neighborhood and $a \in F_s$ and $F_s \subset q^{-1}(V)$. Thus there is $g : \underline{A} \to S$ choosing this $s \in S$ given a. Hence

$$A \subset \bigcup_{a \in \underline{A}} F_{g(a)} \subset q^{-1}(V).$$

Denote the large union by Z_{n+1}. It is open and belongs to $\mathcal{F}$ by the strong regularity axiom. The chain (6) has thus been prolonged successfully.

By dependent choice we may form

$$Z_\omega = \bigcup_{n=0}^{\infty} Z_n.$$

Clearly it is a subset of $q^{-1}(V)$ and it contains x. Since the chain of sets Z_n is increasing, and every Z_n is open in X for $n \geq 1$, it clear that Z_ω is open as well. Moreover, $Z_\omega \in \mathcal{F}$ as $\mathcal{F}$ is $\mathbb{N}$-closed. Finally, $\widetilde{Z_\omega} = Z_\omega$ since $\widetilde{Z_n} \subset Z_{n+1}$ for each n. By definition of T there is some $t \in T$ with $Z_\omega = F_t$ and $q(x) \in q[F_t] \subset V$.

Note that for $V = q[F_{s_1}] \cap q[F_{s_2}]$ this shows $\mathcal{F}$ to be a neighborhood base as well. □

3 A construction of quotient topology in constructive set theory

In topology one often defines abstractly a *quotient map* to be a continuous, onto map $f : X \to Y$ which is such that, for any topological space Z, a function $h : Y \to Z$ is continuous iff $h \circ f$ is continuous. In this case Y is said to have the *quotient topology*. The map q of (1) satisfies this condition. Also given any onto function $f : X \to Y$ we may define an equivalence relation $\sim_f$ by

$$x \sim_f y \Longrightarrow f(x) = f(y).$$

Then $Y \cong X/{\sim_f}$, as sets, and the quotient topology on $X/{\sim_f}$ determines a topology on Y so that f is a quotient map.

Definition 3.1. A *neighborhood space* is a pair (X, τ) of a set X and a set $\tau \subset Pow(X)$ such that

NS1 $\forall x \in X \exists u \in \tau (x \in u)$,

NS2 $\forall x \in X \forall u \in \tau \forall v \in \tau [x \in u \cap v \Rightarrow \exists w \in \tau (x \in w \subset u \cap v)]$.

We say that τ is an *open base* on X.

Definition 3.2. Let (X, τ) be a neighborhood space. Then a set $a \subset X$ is *open* if

$$\forall x \in a \exists u \in \tau (x \in u \subset a).$$

Note that $\emptyset$, X and $u \in \tau$ are open, if a and b are open, then $a \cap b$ is open, and if U is a set of open sets, then $\bigcup U$ is open.

Definition 3.3. A function f between neighborhood spaces (X, τ) and (Y, τ') is *continuous* if

$$\forall x \in X \forall v \in \tau' [f(x) \in v \Rightarrow \exists u \in \tau (x \in u \subset f^{-1}(v))].$$

Note that f is a continuous if and only if $f^{-1}(a)$ is open for all open sets a.

In the following, we assume that (X, τ) is a neighborhood space, Y a set, and f is a function from X onto Y. For $a \subset X$, let

$$\begin{aligned} r \in \mathrm{cov}(a) &\Leftrightarrow r \in \mathrm{mv}(a, \tau) \wedge \forall x \in a \forall u \in \tau ((x, u) \in r \Rightarrow x \in u), \\ \mathrm{Full}_{\mathrm{cov}}(a, c) &\Leftrightarrow c \subset \mathrm{cov}(a) \wedge \forall r \in \mathrm{cov}(a) \exists s \in c (s \subset r), \end{aligned}$$

where $r \in \mathrm{mv}(a, \tau) \overset{\mathrm{def}}{\Leftrightarrow} \forall x \in a \exists u \in \tau ((x, u) \in r)$. Note that if $r \in \mathrm{cov}(a)$, then, letting $\mathrm{ran}(r) = \{u \mid \exists x \in a ((x, u) \in r)\}$, we have $a \subset \bigcup \mathrm{ran}(r)$.

Lemma 3.4. *For each subset a of X, there exists a set c such that* $\mathrm{Full}_{\mathrm{cov}}(a, c)$.

Proof. For each $a \subset X$, by Fullness there exists d such that $d \subset \mathrm{mv}(a, \tau) \wedge \forall r \in \mathrm{mv}(a, \tau) \exists s \in d (s \subset r)$. Letting

$$c = \{r \in d \mid \forall x \in a \forall u \in \tau ((x, u) \in r \Rightarrow x \in u)\},$$

by Restricted Separation, we have $c \subset \mathrm{cov}(a)$. For each $r \in \mathrm{cov}(a)$, since $r \in \mathrm{mv}(a, \tau)$, there exists $s \in \mathrm{mv}(a, \tau)$ such that $s \subset r$, and, since if $(x, u) \in s$ then $(x, u) \in r$ and hence $x \in u$, we have $s \in c$. $\square$

Let $\mathcal{D}$ be a class defined by

$$\mathcal{D} = \left\{(U, c) \mid U \subset \tau \wedge \mathrm{Full}_{\mathrm{cov}} \left(f^{-1} \left(f \left(\bigcup U\right)\right), c\right)\right\}.$$

Note that if $(U, c) \in \mathcal{D}$ and $r \in c$, then $f^{-1}(f(\bigcup U)) \subset \bigcup \mathrm{ran}(r)$.

Proposition 3.5. *There exists a set $D \subset \mathcal{D}$ such that*

1. $\forall u \in \tau \exists c((\{u\}, c) \in D)$,
2. $\forall (U, c) \in D \forall r \in c \exists (U', c') \in D(\mathrm{ran}(r) = U')$.

Proof. Let

$$\phi(A, B) \Leftrightarrow \forall (U, c) \in A \forall r \in c \exists (U', c') \in B \left(\mathrm{ran}(r) = U'\right).$$

We show that $\forall A \subset \mathcal{D} \exists B \subset \mathcal{D} \phi(A, B)$. To this end, suppose that A is a set with $A \subset \mathcal{D}$. Then for each $(U, c) \in A$ and $r \in c$, letting $U' = \mathrm{ran}(r)$ and constructing a set c' such that $\mathrm{Full}_{\mathrm{cov}}\left(f^{-1}\left(f\left(\bigcup U'\right)\right), c'\right)$, by Lemma 3.4, we have $(U', c') \in \mathcal{D}$. Therefore

$$\forall r \in c \exists (U', c') \left((U', c') \in \mathcal{D} \wedge \mathrm{ran}(r) = U'\right),$$

and hence

$$\exists b' \subset \mathcal{D} \forall r \in c \exists (U', c') \in b' \left(\mathrm{ran}(r) = U'\right)$$

by Strong Collection. Since $(U, c) \in A$ is arbitrary, we have

$$\forall (U, c) \in A \exists b' \subset \mathcal{D} \forall r \in c \exists (U', c') \in b' \left(\mathrm{ran}(r) = U'\right),$$

and hence by Strong Collection, there exists a set B' such that

$$\forall (U, c) \in A \exists b' \in B' \left[b' \subset \mathcal{D} \wedge \forall r \in c \exists (U', c') \in b' \left(\mathrm{ran}(r) = U'\right)\right].$$

Thus letting $B = \bigcup B'$, we have $B \subset \mathcal{D}$ and $\phi(A, B)$.

Since $\forall u \in \tau \exists c \mathrm{Full}_{\mathrm{cov}}(f^{-1}(f(u)), c)$, by Lemma 3.4, there exists a set $A_0 \subset \mathcal{D}$ such that $\forall u \in \tau \exists c((\{u\}, c) \in A_0)$ by Strong Collection. Applying **RDC** to $\forall A \subset \mathcal{D} \exists B \subset \mathcal{D} \phi(A, B)$ and A_0, we have a function α with domain ω such that $\alpha(0) = A_0$ and

$$\forall n \in \omega[\alpha(n) \subset \mathcal{D} \wedge \phi(\alpha(n), \alpha(n+1))].$$

Let $D = \bigcup_{n \in \omega} \alpha(n)$. Then it is straightforward to see (1) and (2). $\square$

Using Exponentiation, Restricted Separation and Strong Collection, define sets H and S by

$$\begin{aligned} H &= \{\langle (U_n, c_n) \rangle_{n \in \omega} \in D^\omega \mid \forall n \in \omega \exists r \in c_n \left(\mathrm{ran}(r) = U_{n+1}\right)\}. \\ S &= \left\{\bigcup \bigcup_{n \in \omega} U_n \mid \langle (U_n, c_n) \rangle_{n \in \omega} \in H\right\}. \end{aligned}$$

Lemma 3.6. *For each $a \in S$, a is open and $f^{-1}(f(a)) = a$.*

Proof. Let $a \in S$. Then there exists $\langle (U_n, c_n) \rangle_{n\in\omega} \in H$ such that $a = \bigcup\bigcup_{n\in\omega} U_n$. Suppose that $x \in a$. Then there exist $n \in \omega$ and $u \in U_n \subset \tau$ such that $x \in u \subset \bigcup U_n \subset a$. Therefore a is open.

Suppose that $x \in f^{-1}(f(a))$. Then there exist $n \in \omega$ and $u \in U_n$ such that $x \in f^{-1}(f(u)) \subset f^{-1}(f(\bigcup U_n))$. Since there exists $r \in c_n$ such that $\mathrm{ran}(r) = U_{n+1}$, we have

$$x \in f^{-1}(f(u)) \subset f^{-1}(f(\bigcup U_n)) \subset \bigcup \mathrm{ran}(r) = \bigcup U_{n+1} \subset a.$$

Therefore $f^{-1}(f(a)) \subset a$. □

Proposition 3.7. *Let $u \in \tau$, and let $b \subset X$ be an open set such that $u \subset b$ and $b = f^{-1}(f(b))$. Then there exists $a \in S$ such that $u \subset a \subset b$.*

Proof. Let $D_b = \{(U, c) \in D \mid \bigcup U \subset b\}$. We show that

$$\forall (U, c) \in D_b \exists (U', c') \in D_b \exists s \in c(\mathrm{ran}(s) = U').$$

To this end, suppose that $(U, c) \in D_b$. Define a set

$$r = \{(x, u') \in f^{-1}(f(\bigcup U)) \times \tau \mid x \in u' \subset b\}$$

by Restricted Separation. Then, since b is open, for each $x \in f^{-1}(f(\bigcup U)) \subset f^{-1}(f(b)) = b$, there exists $u' \in \tau$ such that $x \in u' \subset b$, and hence $r \in \mathrm{cov}(f^{-1}(f(\bigcup U)), \tau)$. Therefore, since $\mathrm{Full}_{\mathrm{cov}}(f^{-1}(f(\bigcup U)), c)$, there exists $s \in c$ such that $s \subset r$, and hence there exists $(U', c') \in D$ such that $\mathrm{ran}(s) = U'$, by Proposition 3.5 (2). Since

$$\bigcup U' = \bigcup \mathrm{ran}(s) \subset \bigcup \mathrm{ran}(r) \subset b,$$

we have $(U', c') \in D_b$.

By Proposition 3.5 (1), there exists c such that $(\{u\}, c) \in D_b$. Applying **DC**, we have a function $h : n \mapsto (U_n, c_n)$ with domain ω and range D_b such that $(U_0, c_0) = (\{u\}, c)$ and $\forall n \in \omega \exists s \in c_n(\mathrm{ran}(s) = U_{n+1})$. Therefore, since $h \in H$, we have $a =_{\mathrm{def}} \bigcup\bigcup_{n\in\omega} U_n \in S$ and $u \subset a \subset b$. □

Theorem 3.8. *There exists an open base $\hat{\tau}$ on Y such that for each neighborhood space (Z, σ) and a function $g : Y \to Z$, g is continuous if and only if $g \circ f$ is continuous.*

Proof. Let

$$\hat{\tau} = \{f(a_1) \cap \ldots \cap f(a_n) \mid a_1, \ldots, a_n \in S, n \geq 0\}).$$

Then $\hat{\tau}$ clearly satisfies (NS1) and (NS2).

Let (Z, σ) be a neighborhood space and g a function form Y to Z. Suppose that g is continuous. Let $x \in X$ and $w \in \sigma$ with $g(f(x)) \in w$. Then, since g is continuous, there exists $v \in \hat{\tau}$ such that $f(x) \in v \subset g^{-1}(w)$. Since $v = f(a_1) \cap \ldots \cap f(a_n)$ for some $a_1, \ldots, a_n \in S$ and $n \geq 0$, we have

$$\begin{aligned} x \in f^{-1}(v) &= f^{-1}(f(a_1) \cap \ldots \cap f(a_n)) \\ &= f^{-1}(f(a_1)) \cap \ldots \cap f^{-1}(f(a_n)) \\ &= a_1 \cap \ldots \cap a_n \subset f^{-1}(g^{-1}(w)). \end{aligned}$$

Therefore, since $a_1 \cap \ldots \cap a_n$ is open, there exists $u \in \tau$ such that $x \in u \subset a_1 \cap \ldots \cap a_n \subset f^{-1}(g^{-1}(w))$. Thus $g \circ f$ is continuous.

Conversely suppose that $g \circ f$ is continuous. Let $y \in Y$ and $w \in \sigma$ with $g(y) \in w$. Then, since f is surjective, there exists $x \in X$ such that $y = f(x)$, that is $g(f(x)) \in w$. Therefore since $g \circ f$ is continuous, there exists $u \in \tau$ such that $x \in u \subset b =_{\text{def}} f^{-1}(g^{-1}(w))$. Noting that b is open and $f^{-1}(f(b)) = b$, there exists $a \in S$ such that $u \subset a \subset b$, by Proposition 3.7, and hence $f(a) \in \hat{\tau}$ and $y = f(x) \in f(u) \subset f(a) \subset f(b) = g^{-1}(w)$. Thus g is continuous. □

4 Final topologies

A generalisation of the quotient topology is that of a *final topology* [4] (which is sometimes called a *coinduced topology*).

Let X be a set. Let (Y_i, τ_i), $i \in I$, be a family of neighborhood spaces and let $f_i : Y_i \to X$, $i \in I$, be a family of functions. An open base σ on X is *final* for the family (f_i) if for any function $g : X \to Z$ and any open base η on Z, $g : (X, \sigma) \to (Z, \eta)$ is continuous iff for each i, $g \circ f_i : (Y_i, \tau_i) \to (Z, \eta)$ is continuous. Note that if σ' is another final open base on X for the same family of functions, then the identity function $(X, \sigma) \to (X, \sigma')$ is a homeomorphism, and hence the neighborhood bases give the same open sets, i.e. have the same topology.

Example 4.1. For an onto map $f : Y \to X$, and an open base τ on Y, the final topology on X for f is the quotient topology.

Next we show that final topologies for disjoint sums exists.

Lemma 4.2. *Suppose that (Y_i, τ_i), $i \in I$, is a family of neighborhood spaces. Let*

$$X = \dot{\cup}_{i \in I} Y_i =_{\text{def}} \{(i, y) : i \in I, y \in Y_i\},$$

and let the canonical injections $\kappa_i : Y_i \to X$ be given by $\kappa_i(y) = (i, y)$. Then there is a final open base σ on X for the family (κ_i).

Proof. If τ_i is the open base $\{U_{ij}\}_{j \in J_i}$, then σ may be constructed as

$$\{\{i\} \times U_{ij}\}_{(i,j) \in S}$$

where $S = \{(i, j) : i \in I, j \in J_i\}$. We leave the verification that this is indeed a final open base for $(\kappa_i)_i$ to the reader.

□

We can now prove the general existence theorem for final topologies.

Theorem 4.3. *Let X be a set. Let (Y_i, τ_i), $i \in I$, be a family of neighborhood spaces and let $f_i : Y_i \to X$ be a family of functions. Then there exists a final open base ϕ on X for the family $(f_i)_i$.*

Proof. The base ϕ will be constructed as quotient topology of a certain sum topology.

Equip X with the discrete open base δ_X. Then form a new index set $J = I \,\dot{\cup}\, \{\star\} = \{(0, i) : i \in I\} \cup \{(1, \star)\}$, and define a new family of neighborhood spaces A_j, $j \in J$, by

$$A_{(0,i)} = (Y_i, \tau_i), \qquad A_{(1,\star)} = (X, \delta_X).$$

Form the sum neighborhood space as in Lemma 4.2,

$$S = (\dot{\cup}_{j \in J} A_j, \sigma)$$

and where $\kappa_j(x) = (j, x)$. Next, let $\sim$ be the smallest equivalence relation on S so that

$$((0, i), y) \sim ((1, \star), f_i(y))$$

for all $i \in I$ and $y \in A_i$. Form the quotient topology $(S/\!\sim, \rho)$ with respect to this equivalence relation, and write $q : S \to S/\!\sim$ for the quotient map. We remark that ρ is a final open base on $S/\!\sim$ for the family $(q \circ \kappa_j)_j$. The function $h : S \to X$ defined by $h((0, i), y) = f_i(y)$ and $h((1, \star), y) = y$ gives a bijection of sets $\overline{h} : (S/\sim) \to X$. Thus there is an open base ρ' on X so that $\overline{h}$ is a homeomorphism. We let $\phi = \rho'$ and verify that it is final for the family (f_i).

Consider the composition $t_j = \overline{h} \circ q \circ \kappa_j$. It is clearly continuous, and for $j = (0, i)$

$$t_j(x) = \overline{h}(q(j, x)) = h(j, x) = f_i(x).$$

Hence f_i is continuous.

Let $g : X \to Z$ be a function. Let η be an open base on Z. Suppose first that $g : (X, \phi) \to (Z, \eta)$ is continuous. Since f_i is continuous, $g \circ f_i$ is indeed continuous.

Conversely, suppose that $g \circ f_i$ is continuous for each $i \in I$. Since $f_i = t_{(0,i)}$, also $g \circ t_{(0,i)}$ is continuous for $i \in I$. The domain of $g \circ t_{(1,\star)}$ has the discrete topology, so $g \circ t_{(1,\star)}$ is trivially continuous. Hence $g \circ t_j$ is continuous for all $j \in J$. The open base ρ' on X is final for the family $(t_j)_{j \in J}$, so g is continuous. □

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