Symposium on constructivity and computability in algebra, analysis, logic and topology

Uppsala, June 9-10, 2011

Thursday 9 June, lecture hall Å4007, Ångström laboratory

9.30-10.20	Dag Normann	Banach spaces as data types
10.35-11.25	Dirk Pattinson	Continuous functions as algebraic data types
11.40-12.30	Per Martin-Löf	Path models of type theory

Lunch break

14.15-15.05	Warwick Tucker	Validated numerics - a short introduction to rigorous computations
15.20-16.10	Peter Schuster	Induction in algebra

Coffee break

16.40-17.30	Thorsten Altenkirch	The partiality monad

Friday 10 June, Å4007

9.30-10.20	Sara Negri	Constructive embeddings of intemediate logics
10.35-11.25	Josef Berger	On Dickson's lemma
11.40-12.30	Iosif Petrakis	Extension of Scott's information systems to regular and T_1 spaces

Lunch break

14.15-15.05	Jan von Plato	Classical natural deduction
15.20-15.50	Erik Palmgren	Nonstandard analysis and Brouwer's continuity principle
16.05-16.55	Hajime Ishihara	Supergeometric theories and set-generated classes

Organizer: Erik Palmgren, Department of Mathematics, Uppsala University.

Speakers and abstracts

Thorsten Altenkirch, University of Nottingham

The Partiality Monad

Abstract: We introduce the partiality monad which is definable in Type Theory with quotient types (corresponding to a pretopos). The partiality monad can be used to model partial computations within total type theory. We can define a fixpoint combinator but this depends on the requirement that the input functional is continuous, which is true for all definable functions. Can we work in a type theory where this is provable?

Thierry Coquand, Chalmers, Göteborg

Constructive finite free resolutions[1]

Abstract: Homological algebra was introduced by Hilbert in his 1890 paper "Ueber die Theorie der algebraischen Formen" (the paper where Hilbert proved the finite basis theorem). Homological algebra can be described as linear algebra over an arbitrary ring and one would expect most statements to be first-order. This is not the case since most treatments assume Noetherian hypotheses. An exception is the book of Northcott on Finite Free Resolutions. However even in this book, most proofs use non effective existence of prime ideals or minimal prime ideals. We present direct constructive (and first-order) proofs for most of the result in Northcott's book. In particular, we give a constructive proof of the fact that a regular ring (ring such that any finitely generated ideal has a finite free resolution) is a gcd domain.

Hajime Ishihara, Japan Advanced Institute of Science and Technology

Supergeometric theories and set-generated classes

Abstract: We introduce supergeometric theories over a set S and their models which are subsets of S. Then we show that the class $\mathfrak{M} \subseteq \mathrm{Pow}(S)$ of a supergeometric theory is set-generated in **CZF**, that is, there exists a set $M \subseteq \mathfrak{M}$ such that $\alpha = \bigcup\{\beta \in M \mid \beta \subseteq \alpha\}$ for each $\alpha \in \mathfrak{M}$. We also present some applications to algebra and formal topology.

[1] Only presented by abstract

Per Martin-Löf, Stockholm University

Path models of type theory

Sara Negri, University of Helsinki

Constructive embeddings of intemediate logics

Abstract: Using labelled formulae, a cut-free sequent calculus for intuitionistic propositional logic is presented, together with an easy cut-admissibility proof; both extend to cover, in a uniform fashion, all intermediate logics characterised by frames satisfying conditions expressible by one or more geometric implications. Each of these logics is embedded by the Gödel-McKinsey-Tarski translation into an extension of S4. Both soundness and faithfulness of the embedding are proved in a simple and general way by constructive proof-theoretic methods. [Joint work with Roy Dyckhoff].

Dag Normann, University of Oslo

Banach spaces as data types

Erik Palmgren, Uppsala University

Nonstandard analysis and Brouwer's continuity principle

Dirk Pattinson, Imperial College

Continuous functions as algebraic data types

Abstract: In previous work, we have exhibited a representation of continuous stream functions as mixed inductive / coinductive data types. Here, the inductive part yields continuity (outputs depend on finite input prefixes) whereas the coinductive part gives productivity (the computation never ends). In this talk, we investigate generalisations of this basic pattern with the goal giving a representation theorem for continuous higher-order functions.

Iosif Petrakis, Universtität München.

Extension of Scott's information systems to regular and T_1 spaces.

Jan von Plato, University of Helsinki

Classical natural deduction

Abstract: In a step of indirect inference in natural deduction, a negative temporary assumption -A is closed and A concluded. When assumptions are closed otherwise, they are subformulas of the conclusion, as in implication introduction, or of the major premiss, as in those elimination rules that close assumptions, and the subformula property of normal derivations follows. The situation is different with indirect proof, because the conclusion A can be a major premiss in an elimination, and it can be lost trace of even in the absence of non-normal instances of the rest of the rules. Solutions to the problem have included the leaving out of disjunction and existence from the language and the restriction of conclusions of indirect proof to atomic formulas, by which they cannot be major premisses of elimination rules. Other solutions involve restrictions in the way elimination rules can be instantiated, and yet others contain global proof transformations.

In the present work, it is shown that derivations in the full language of predicate logic can be so transformed by standard methods of local permutations that no conclusion of an indirect step of proof is the major premiss of an elimination rule. For the rest, normal form is defined as for intuitionistic derivations, in particular, no a priori restrictions are imposed on rule instances: Normal derivations and whatever forms of rule instances they may contain come out purely as results of a normalization procedure. It follows in particular that normal derivations have the subformula property. The situation is particularly clear if natural deduction is formulated in terms of general elimination rules (cf. von Plato 2001) with the definition: A derivation is normal if all major premisses of elimination rules are assumptions. This definition can be applied directly to classical natural deduction for predicate logic.

Peter Schuster, University of Leeds and Universität München

Induction in Algebra

Abstract: Many a concrete theorem of abstract algebra admits a short and elegant proof by contradiction but with Zorn's Lemma (ZL). A few of these theorems have recently turned out to follow in a direct and elementary way from the Principle of Open Induction distinguished by Raoult. A proof of the

latter kind may be obtained systematically from a proof of the former sort, and the tree one can grow alongside the induction encodes the computation corresponding to the theorem. If the theorem happens to have finite input data, then a finite partial order carries the required instance of induction, which thus is constructively provable. The ideal objects characteristic of any invocation of ZL are eliminated, and it is made possible to pass from classical to intuitionistic logic. This approach is intended as a contribution to a partial realisation of Hilbert's Programme, and was motivated by related work of Berger, Coquand and by the rise of dynamical and logical approaches to algebra.

Warwick Tucker, Uppsala University

Validated Numerics - a short introduction to rigorous computations

Abstract: We will present an efficient means of performing numerical computations with rigorous error bounds. The basic idea is to use set-valued mathematics as the underlying framework. This enables us to change focus from approximating the solution to enclosing the same. We will apply these techniques to several problems, ranging from simple root-finding and quadrature to Hilbert's 16th problem.