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ÖVNINGAR 6
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Exercises 6

1. Use the method from the proof of the completeness theorem (see Sigstam's compendium) to prove or refute the following sequents. P, Q are unary predicate symbols, R is a 3-ary predicate symbol.
   (a) $\forall x\,(P(x) \supset Q(x)) \longrightarrow \exists x\,(P(x) \wedge Q(x))$.
   (b) $\forall x \exists y \forall z\, R(x, y, z) \longrightarrow \exists x \exists y \exists z\, R(x, y, z)$.

2. Argue that if $\varphi \longrightarrow \psi$ is a valid sequent whose both sides only contain the logical operators $\forall, \exists, \wedge$, then the sequent has a proof in the sequent calculus which only use the rules for these operators and logical axioms of the form $C, \Gamma \longrightarrow \Delta, C$. Consider an arbitrary sequent $\varphi \longrightarrow \psi$. Discuss whether there are some natural restrictions on the form of φ and ψ which guarantee that the search process in the proof of the completeness theorem stops after finitely many steps (with a proof of counter model as a result).

3. Let P be a binary predicate symbol. Let φ be the formula

$$\forall x\, \neg P(x, x) \wedge \forall xyz\,(P(x, y) \wedge P(y, z) \rightarrow P(x, z)) \wedge \forall x\, \exists y\, P(x, y).$$

   Show that φ has only infinite models.

4. (a) Let Γ be a set of first order formulas — a theory. Show that if Γ has only finite models, then there is a number n such that every such model has at most n elements. (Hint: the compactness theorem.)
   (b) Draw the conclusion that there is no first order theory Γ whose models are precisely the finite partially ordered sets.

5. A graph is a non-empty set with a reflexive and symmetric relation. The graph is connected if every pair of nodes can be connected by a finite sequence of edges. Let L be the language consisting of the binary relation R. Is there a closed L-formula φ such that for all L-structures $\mathcal{A}$:

$$\mathcal{A} \models \varphi \iff \mathcal{A} \text{ is a connected graph ?}$$

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