UPPSALA UNIVERSITET   EXERCISES 4
Matematiska institutionen   Tillämpad Logik DV1, vt-04
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Exercises and Hand-in Problems 4, Tillämpad Logik DV1

Hand-in problems are

$$\boxed{2, 3, 5, 7, 8, 11}$$

of the exercises below. Solutions should be handed in by Monday, May 17, 2004, clearly written and well explained[1] in Swedish or English. (You may work in groups of three.)

Corrected solutions to Exercise sets 2 – 4 may be fetched at my office 3339, Department of Mathematics. All exercises handed in (on time) are expected to be corrected by May 19.

1. Investigate the truth tables of 2-CNF propositional formulas with three propositonal variables P, Q, R. Is the propositional formula $P \vee Q \vee R$ logically equivalent to any 2-CNF propositional formula?

    Note: there are efficient polynomial time algorithms for checking satisfiability of 2-CNF formulas. None such are known for 3-CNF formulas (unless P=NP). See H. Kleine Büning, T. Lettman: *Propositional logic: deduction and algorithms.* Cambridge University Press 1999.

2. * Using resolution derive the empty clause from the following set of clauses, where c and d are constants and x, y and z are variables.

$$\{F(c) \vee F(d),$$
$$F(c) \vee \neg G(d),$$
$$G(c) \vee F(d),$$
$$G(c) \vee \neg G(d),$$
$$\neg H(c) \vee F(d),$$
$$\neg H(c) \vee \neg G(d),$$
$$\neg F(x) \vee G(x) \vee \neg F(y) \vee H(y),$$
$$\neg F(x) \vee \neg H(x) \vee G(x),$$
$$\neg F(z) \vee \neg G(z) \vee H(z)\}$$

    This set of clauses was obtained via Skolemization and clausification of the formula $A \wedge B \rightarrow C$ on page 57 of Smullyan's book.

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[1]Unsatisfactorily presented solutions may be returned ungraded.

3. Construct, using the systematic tableaux method, a model satisfying the formula
$$A =_{\text{def}} \exists x (P(x) \wedge (\exists y)(R(x,y) \wedge \neg P(y))).$$

Thus put $T\ A$ on the top of the tableau and execute the algorithm given in Smullyan's book. Present the Hintikka set(s) and model(s) emerging from the finished tableau.

Model-checking

4. HR 3.2.1 (= Huth and Ryan, Exercise 3.2.1)

5. HR 3.2.2 (a) – (b)

6. HR 3.3.2

7. HR 3.7.1

Ordered binary decision diagrams

8. HR 6.4.1

9. HR 6.4.3

10. HR 6.5.2

11. HR 6.5.3

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