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EXERCISES 4
Tillämpad Logik, ht-07
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Exercises 4 — Applied Logic

1. Prove that the following type of Martin-Löf type theory is inhabited

$$P =_{\mathrm{def}} (\Pi z : (\Sigma x : A)B)C \to (\Pi x : A)(\Pi y : B)C[(\mathsf{pair}\, x\, y)/z].$$

   That is, find a term a so that $\vdash a : P$ is derivable in Martin-Löf type theory as presented in Chapter 1 of (Coquand *et al.* 2007). Here it is assumed that $\vdash A$ type, $x : A \vdash B$ type and $z : (\Sigma x : A)B \vdash C$ type

2. *Propositions-as-types.* Translate the following logical formulas or propositions into types of Martin-Löf type theory assuming quantifiers range over the natural numbers

   (a) $A \supset A \vee B$

   (b) $A \wedge B \supset B$

   (c) $(A \wedge B \supset C) \supset A \supset (B \supset C)$.

   (d) $(\exists y)A(y) \vee (\exists y)B(y) \supset (\exists y)(A(y) \vee B(y))$

   (e) $(\forall y)A(y) \supset (\forall y)(A(y) \supset B(y)) \supset (\forall y)B(y)$.

   In case (d) and (e) find a term that inhabits the type. Compare to your solution of Exercise 3.2.

3. (*) *Type-theoretic "axiom" of choice.* Prove that

$$(\Pi x : A)(\Sigma y : B)C \to (\Sigma f : (A \to B))(\Pi x : A)C[(f\, x)/y]$$

   is inhabited. Here it is presupposed that $\vdash A$ type, $\vdash B$ type and $x : A, y : B \vdash C$ type

4. Give an interpretation of the previous exercise in terms of propositions and truth.

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