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EXERCISES
BONUS PROBLEMS 4
Applied Logic, Fall 2009
2009-10-05

Principles of automated theorem proving

The BONUS PROBLEMS are marked (+) below. Solutions of these are to be handed in at the latest on 26 October. Maximum bonus for this set of problems is 2.5%.

1. (+) Exercise 1.1.1 in the handout [E]: *Equational Logic, Unification and Term Rewriting,* Lecture notes E. Palmgren 2007.

2. Exercise 1.1.2 in [E].

3. (+) Exercise 1.2.1 in [E].

4. Exercise 1.2.2 in [E].

5. Determine the Skolem conjunctive normal form for each of the following formulas, and thereby its equivalent set of clauses. P and Q are predicate symbols.

   (a) (+) $\forall u\,[\forall x\,\neg\exists y\,P(x,y) \rightarrow \neg\forall x\,(\exists z\,P(x,y) \rightarrow \forall y\,P(u,y))]$.

   (b) $\forall x\,(\exists y\,P(x,y) \leftrightarrow \exists z\,Q(z,x))$.

6. Deduce using resolution the empty clause ($\Box$) from the following set of clauses

   (a) $\{\neg P(x), P(x) \vee \neg Q(x), P(x) \vee \neg R(x), Q(x) \vee R(x)\}$,

   (b) (+) $\{\neg P(x,y), P(x,y) \vee \neg Q(x,z) \vee \neg P(z,y), P(x,y) \vee \neg Q(x,y), Q(a,b), Q(b,c)\}$.

7. Deduce using resolution the empty clause ($\Box$) from the following set of clauses

   $$S = \{Q(x) \vee P(f(x)) \vee P(v), R(g(y),y) \vee \neg P(y), \neg R(g(f(z)),u), \neg Q(c)\}.$$

   For what $n \geq 0$ does $\Box \in \mathrm{Res}^n(S)$ hold? (See notation in Das.)

8. (+) Show that the most general unifier of

$$f(g(x_1, x_1), g(x_2, x_2), \ldots, g(x_n, x_n)) \text{ and } f(x_2, x_3, \ldots, x_{n+1})$$

has 2^n occurrences of the variable x_1. (This shows that it is necessary to represent terms in some efficient way, for instance as DAGs). There is a indeed a linear time unification algorithm.[1])

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[1] A. Martelli, U. Montanari: *An efficient unification algorithm.* ACM Transactions on Programming Languages and Systems 4(1982), 258 – 281.