Principles of automated theorem proving

The BONUS PROBLEMS are marked (+) below. Solutions of these are to be handed in at the latest on 26 October. Maximum bonus for this set of problems is 2.5%.


2. Exercise 1.1.2 in [E].

3. (+) Exercise 1.2.1 in [E].

4. Exercise 1.2.2 in [E].

5. Determine the Skolem conjunctive normal form for each of the following formulas, and thereby its equivalent set of clauses. $P$ and $Q$ are predicate symbols.

   (a) (+) $\forall u[\forall x \neg \exists y P(x, y) \rightarrow \forall x (\exists z P(x, y) \rightarrow \forall y P(u, y))]$.

   (b) $\forall x (\exists y P(x, y) \leftrightarrow \exists z Q(z, x))$.

6. Deduce using resolution the empty clause ($\square$) from the following set of clauses

   (a) $\{-P(x), P(x) \lor \neg Q(x), P(x) \lor \neg R(x), Q(x) \lor R(x)\}$,

   (b) (+) $\{-P(x, y), P(x, y) \lor \neg Q(x, z) \lor \neg Q(z, y), P(x, y) \lor \neg Q(x, y), Q(a, b), Q(b, c)\}$.

7. Deduce using resolution the empty clause ($\square$) from the following set of clauses

   $S = \{Q(x) \lor P(f(x)) \lor P(v), R(g(y), y) \lor \neg P(y), \neg R(g(f(z)), u), \neg Q(c)\}$.

   For what $n \geq 0$ does $\square \in \text{Res}^n(S)$ hold? (See notation in Das.)

8. (+) Show that the most general unifier of
\[ f(g(x_1, x_1), g(x_2, x_2), \ldots, g(x_n, x_n)) \text{ and } f(x_2, x_3, \ldots, x_{n+1}) \]

has \(2^n\) occurrences of the variable \(x_1\). (This shows that it is necessary to represent terms in some efficient way, for instance as DAGs). There is indeed a linear time unification algorithm.\(^1\)