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LABORATION 2
Tillämpad logik DV1, ht-01
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Laboration 2

The exercises may be solved in groups of up to three persons. The solved Alfa-files (`.alfa`) should be sent by e-mail to `palmgren@math.uu.se` at the *latest October 17, 2001.* (If sent in before October 10, 2001, you get an extra chance to amend and correct your exercises.) Also hand in an annotated and commented version on paper (or insert the comments directly in the Alfa-file, if you know how to do this without breaking the file). The files necessary for the laboration can be downloaded from the home page of the course. Put them in an empty directory and make this your working directory. Start Alfa using the command `alfa`.

Dont forget to do the termination check for the proofs you submit. (Mark the definition(s) and press the Check termination option in the menu.) Beware that the system sometimes suggest circular proofs!

1. In this exercise you are going to give a constructive proof of that the maximum of two natural numbers exists, and from this proof extract a function computing the maximum. What is required to prove is

   $$(\forall m \in \mathbb{N})(\forall n \in \mathbb{N})(\exists z \in \mathbb{N})[(m \leq n \wedge z = n) \vee (n \leq m \wedge z = m)],$$

   or rather its translation in type theory. The relation $\leq$ (`Leq`) is defined inductively

   $$\begin{aligned} (0 \leq \mathsf{S}(n)) &\equiv N_1 \\ (\mathsf{S}(m) \leq 0) &\equiv \emptyset \\ (\mathsf{S}(m) \leq \mathsf{S}(n)) &\equiv (m \leq n) \end{aligned}$$

   and then equality (`Eq`) is defined by $(m = n) \equiv (m \leq n) \wedge (n \leq m)$.

   The exercise is prepared in the file `lab2.alfa`. Fill in all question marks.

2. (optional, for purists) Construct `maxproof` using the recursion operator instead of the `case` construction.

3. (optional, gives 1-2 bonus point) Define in Alfa/Agda a type constructor `List :: Set -> Set` which to each type A in `Set` assigns the type of lists of elements of A. Define introduction, elimination and computation rules. If $\leq_A$ is a linear order on A (such as $\leq$ is on $\mathbb{N}$), define a suitable lexicographic order $\leq_{\texttt{List}(A)}$ on `List`(A). Prove that your lexicographic order is decidable, if $\leq_A$ is decidable. Using the decision procedure for $\leq_{\mathbb{N}}$, obtain thus a decision procedure for $\leq_{\texttt{List}(\mathbb{N})}$. Iterate to obtain a decision procedure for lists of lists of numbers.

Exercise 1 is a modification of an exercise due to Lars Lindqvist.